NUMERICAL SIMULATION OF THERMOFORMING

M. K. Warby and J. R. Whiteman
Brunel Institute of Computational Mathematics
Brunel University, Uxbridge Middlesex UB8 3PH
Mike.Warby@brunel.ac.uk, John.Whiteman@brunel.ac.uk
Tel: ++44 1895 274000 ext. 2168/2169, Fax: ++44 1895 203303

Numerical simulations of the large constrained deformation of polymeric structures frequently involves, at some stage, geometrical situations which are difficult to model. These are in addition to any difficulties encountered in the description and in the implementation of complicated constitutive relations. In the case of thermoforming the difficult geometrical situations occur as thin polymeric sheets are deformed into complicated mould shapes to form container structures. In fact in the simplest implementation of thermoforming which involves pressure only forcing action the deformation is essentially 'geometry driven' and as a consequence the geometrical difficulties are the main difficulties in the problem. Geometrical quantities, specifically the thickness distribution, are also among the main quantities of interest.

To construct a model of thermoforming the set up is usually as follows. The thin sheet is modelled as a membrane which has the effect of reducing a 3D problem to a more manageable 2D problem. The membrane is assumed to stick on contact with the mould which corresponds to the warm sheet effectively freezing to the cool mould surface.

With $\underline{e}_1$, $\underline{e}_2$ and $\underline{e}_3$ as the usual Cartesian base vectors the deformation of the mid-surface is described by

$$x_1\underline{e}_1 + x_2\underline{e}_2 \to \underline{w} := (x_1+u_1)\underline{e}_1 + (x_2+u_2)\underline{e}_2 + u_3\underline{e}_3,$$

$\forall (x_1,x_2) \in \Omega$, where $\underline{u} = (u_i)$ is the displacement. With $\boldsymbol{F}$ denoting the deformation gradient, $\boldsymbol{F} = \boldsymbol{RU} = \boldsymbol{VR}$ being the polar decomposition, $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$ and $\boldsymbol{B} = \boldsymbol{FF}^T = \boldsymbol{R}^T\boldsymbol{CR}$ denoting the right and left Cauchy Green deformation tensors and with $\boldsymbol{\sigma}$ denoting the Cauchy Stress tensor the membrane assumption implies that for an incompressible material

$$\boldsymbol{C} = \begin{pmatrix} C_{11} & C_{21} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & \lambda^2 \end{pmatrix}, \quad \boldsymbol{R}^T\boldsymbol{\sigma}\boldsymbol{R} = \begin{pmatrix} \tilde{\sigma}_{11} & \tilde{\sigma}_{21} & 0 \\ \tilde{\sigma}_{21} & \tilde{\sigma}_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where $\lambda^2(C_{11}C_{22} - C_{21}^2) = 1$. With λ_1, λ_2 and $\lambda_3 = \lambda$ denoting the principal values (eigenvalues) of $\boldsymbol{C}$ and with σ_1 and σ_2 denoting the non-zero principal stresses, the stress-stretch relation for an isotropic hyperelastic material with strain energy function W can be expressed in the form

$$\sigma_i = \lambda_i \frac{\partial W}{\partial \lambda_i}, \quad i = 1, 2.$$

Although the deformation in thermoforming occurs quickly, the situation is still essentially quasi-static. At any given stage, when the applied pressure is P, the equilibrium of the part of the sheet $\Omega_{fr}(P)$ not yet stuck to the mould is described by

$$\iint_{\Omega_{fr}} h_0(\boldsymbol{\Pi}^T : \nabla \underline{v}) - P\, g(\underline{u}, \underline{v})\ dx_1 dx_2 = 0,$$

$\forall$appropriate $\underline{v}$, where $\boldsymbol{\Pi} = \boldsymbol{F}^{-1}\boldsymbol{\sigma}$ is the nominal stress tensor and where

$$3g = \underline{v} \cdot \left(\frac{\partial \underline{w}}{\partial x_1} \times \frac{\partial \underline{w}}{\partial x_2}\right) + \underline{w} \cdot \left(\frac{\partial \underline{v}}{\partial x_1} \times \frac{\partial \underline{w}}{\partial x_2}\right) + \underline{w} \cdot \left(\frac{\partial \underline{w}}{\partial x_1} \times \frac{\partial \underline{v}}{\partial x_2}\right).$$

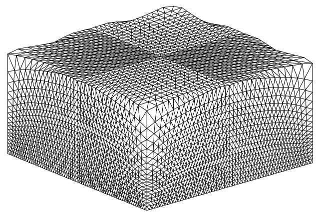
In a corresponding finite element spatial discretization we take an increasing sequence of discrete pressures $P_0 < P_1 < P_2 < \cdots$ and for each P_j we construct a triangular mesh $\Omega^h(P_j)$ of Ω from which a subset of the triangles gives us a triangular mesh $\Omega^h_{fr}(P_j)$ of the region not yet stuck to the mould. We then have the following steps.

1. We attempt to solve the finite element equations of quasi-static equilibrium, i.e. a nonlinear system $\underline{f}(\underline{a}; P_j) = \underline{0}$, where $\underline{a}$ consists of all the unknown nodal parameters.

2. We constrain any points which go outside of the mould.

3. If any points have been constrained we then possibly re-mesh the new 'free' region and attempt to solve the new nonlinear system $\underline{f}(\underline{a}; P_j) = \underline{0}$.

With appropriate robust and reasonably efficient nonlinear solution techniques this procedure works quite well for some mould geometries but leads to modelling difficulties for others. For example, when it works, deforming a square flat sheet $-1 \le x_1, x_2 \le 1$ into a box $-1 \le x_1, x_2 \le 1$, $0 \le x_3 \le 1$ can typically be implemented on a Pentium PC with a P100 processor in about 2 minutes when a uniform mesh of 1024 elements is used for the undeformed domain Ω. The final deformed shape is shown in the figure.



In the figure the large triangles indicates that large in plane stretching has occurred and consequently these regions correspond to where the structure will be the most thin. Such regions do not however correspond to where the modelling difficulties are encountered in other geometries. In fact the difficulties relate to the more densely packed triangles where, depending on the geometry, compression can occur with $\min(\lambda_1, \lambda_2) < \lambda_3 = \lambda$ and with $\min(\sigma_1, \sigma_2) < 0$. When this occurs the membrane can 'wrinkle' and 'fold' and we can fail to satisfactorily complete the simulation. This compression effect can of course happen in practice although we believe that it does not happen in all the cases that the simulation fails suggesting that in certain areas of the sheet the membrane model needs to be modified to allow for some resistance to bending. This idea is being pursued in attempt to increase the robustness of the computational model.

References

[1] H.F. Nied, C. A. Taylor, and H. G. deLorenzi. Three dimensional finite element simulation of thermoforming. *Polymer Engineering and Science*, 30, No. 20:1314—1322, 1990.