Mode-Field Diameter (MFD)

- Important parameter determined from mode-field distribution of fundamental LP₀₁ mode.
- Characterized by various models
- Main consideration: how to approximate the electric field distribution
- Gaussian distribution

$$E(r) = E_0 e^{\frac{r^2}{W^2}}$$

 MDF width 2W₀ of the LP₀₁ mode can be defined as:

$$2W_{0} = 2\left[\frac{\int_{0}^{\infty} r^{3}E^{2}(r)dr}{\int_{0}^{\infty} rE^{2}(r)dr}\right]^{1/2}$$





where

- •E(r) denotes the field distribution of the LP₀₁ mode
- •r is the radius
- • W_0 is the width of the electric field distribution

Power law for refractive-index variation

$$n(r) = \begin{cases} n_1 \left[1 - 2\Delta \left(\frac{r}{a}\right)^{\alpha} \right]^{1/2} & 0 \le r \le a \\ n_1 (1 - 2\Delta)^{1/2} \approx n_1 (1 - \Delta) = n_2 & r \ge a \end{cases}$$

where

- •a is the core radius
- • n_1 is the refractive index of the core
- • n_2 is the refractive index of the cladding
- • α defines the shape of the index profile

Index difference for gradient index fibre

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1}$$

 Determining the NA for graded-index fibres is more complex than for step index fibres, since it is a function of position across the core end face. Geometrical optics considerations show that light incident on the fibre core at position r will propagate as a guided mode only if it is within the local numerical aperture NA(r) at that point

Local numerical aperture

$$NA(r) = \begin{cases} \left[n^2(r) - n_2^2 \right]^{1/2} \approx NA(0)\sqrt{1 - (r/a)^{\alpha}} & r \le a \\ 0 & r > a \end{cases}$$

where the Axial numerical aperture is defined as
$$NA(0) = \left[n^2(0) - n_2^2 \right]^{1/2} = \left[n_1^2 - n_2^2 \right]^{1/2} \approx n_1 \sqrt{2\Delta}$$

the NA of a graded-index fibre decrease from NA(0) to zero as r moves from the fibre axis to the core-cladding boundary

The number of bound modes in a graded-index fibre is

$$M = \frac{\alpha}{\alpha + 2} \alpha^2 k^2 n^2 \Delta$$

 A comparison of the numerical apertures for fibres having various α profile



Signal degradation in optical fibres

- Two main sources of degradation
- Attenuation (fibre loss of signal loss)
- Dispersion (signal distortions)

Attenuation

• Power at a distance z :

$$P(z)=P(0)e^{-\alpha_p z}$$

where

$$\alpha_{p} = \frac{1}{z} \ln \left[\frac{P(0)}{P(z)} \right]$$

is the attenuation coefficient (in km⁻¹)

Attenuation

•The common procedure for calculating optical signal attenuation in a fibre is to express the attenuation coefficient in unit of decibels per kilometre, denoted by dB/Km

$$\alpha(dB / km) = \frac{10}{z} \log \left[\frac{P(0)}{P(z)}\right] = 4.343 \alpha_p(km^{-1})$$

This parameter is referred to as the fibre loss or the fibre attenuation

Examples

Example: An ideal filter would have no loss so that $P_{out} = P_{in}$. This corresponds to a 0 db/km attenuation, which, in practice, is impossible. An actual low-pass fibre may have a 3 dB/km average loss at 900 nm, for example. This means that the optical signal power would decrease by 50 % over a 1 km length and would decrease by 75 % (a 6 dB loss) over 2 km length, since loss contribution expressed in decibels are additive.

Examples

Example: Consider a 30-Km long optical fibre that has an attenuation of 0.8 dB/Km @ 1300 nm. Suppose we want to find the optical output power P_{out} if 200 μ W of optical power is launched into the fibre. We first express the input power in dBm units:

$$P_{in}(dBm) = 10 \log \left[\frac{P_{in}(W)}{1mW} \right] = 10 \log \left[\frac{200 \times 10^{-6}W}{1 \times 10^{-3}W} \right] = -7.0 dBm$$

The output power level (in dBm) at zs=30 Km is

$$P_{out}(dBm) = 10 \log \left[\frac{P_{out}(W)}{1mW}\right] = 10 \log \left[\frac{P_{in}(W)}{1mW}\right] - \alpha z = -7.0 dBm - (0.8 dB / Km)(30 Km) = -31.0 dBm$$

Absorption

- Caused by three mechanisms
 - Absorption by atomic defects in the glass composition
 - Extrinsic absorption by impurity atoms in the glass material
 - Intrinsic absorption by basic constituent atoms of the fibre material

Absorption by atomic defects

•Atomic defects are imperfections in the atomic structure of the fibre material. Examples are missing molecules, high-density clusters of atom groups, or oxygen defects in the glass structure. Usually, absorption losses arising from these defects are negligible compared with intrinsic and impurity absorption effects. However, they can be significant if the fibre is exposed to ionizing radiation.



The basic response of a fibre to ionizing radiation is an increase in attenuation owing to the creation of atomic defects that absorb optical energy. The higher the radiation level, the larger is the attenuation. However, the attenuation centre will relax or anneal out with time.

Extrinsic absorption by impurity atoms

•Impurity absorption results predominantly from transition metal ions, such as iron, chromium, cobalt, and copper, and from OH (water) ions. The transition metal impurities which are present in the starting materials used for direct-melt fibres range between 1 and 10 parts per billion (ppb), causing losses from 1 to 10 dB/Km.



The peaks and valleys in the attenuation curve resulted in the designation of various "transmission windows" to optical fibres. By reducing the residual OH contents of fibres to around 1 ppb, standard commercially available single-mode fibres have nominal attenuation of 0.5 dB/Km in the 1300-nm window and 0.3 dB/Km in the 1550-nm window. The AllWave® fibre made by Lucent Technologies* is obtained by an effectively complete elimination of water molecules.

*Now see OFS AllWave "Zero Water Peak" SM fibre: http://www.ofsoptics.com

OFS AllWave Fibre

No water peak around 1400nm – gains about 100nm in spectrum

Transmission Characteristics:

Attenuation (uncabled fiber):

Wavelength (nm)	Attenuation (dB/km)	
	Maximum	Typical
1310	0.34	0.32
1383	0.31	0.28
1490	0.24	0.21
1550	0.21	0.19
1625	0.24	0.20
Attenuation vs. Wavelen	gth:	
Range (nm)	Reference(nm) λ	α
1285 - 1330	1310	0.03
1360 - 1480	1385	± 0.04
1525 - 1575	1550	0.02
1460 - 1625	1550	0.04

The attenuation in a given wavelength range does not exceed the attentuation of the reference wavelength(λ)by more than the value α .

Change in Attenuation at Water Peak:

The uncabled fiber attenuation coefficient at the OH⁻ absorption peak (1383 \pm 3 nm) after exposure to hydrogen is \leq 0.31 dB/km and \leq 0.28 dB/km typically. This test simulates long-term hydrogen aging in installed cables.

Chromatic Dispersion:			
Zero dispersion wavelength (λ_0) :	1302 – 1322nm		
Typical zero dispersion wavelength:	1312 nm		
The maximum dispersion slope (S_0) at λ_0 :	0.090 ps/nm ² -km		
Typical dispersion slope:	0.087 ps/nm ² -km		
Mode Field Diameter:			
at 1310 nm	$9.2\pm0.4~\mu m$		
at 1550 nm	$10.4\pm0.5~\mu m$		
Cutoff Wavelength:			
Cable Cutoff Wavelength (λ_{cc}):	<u>≤</u> 1260 nm		

Figures from OFS website

Intrinsic absorption by basic constituent atoms

•Intrinsic absorption is associated with the basic fibre material (e.g., pure SiO_2) and is the principal physical factor that defines the transparency window of a material over a specified spectral region. Intrinsic absorption results from electronic absorption bands in the ultraviolet region and from atomic vibration bands in the near-infrared region.

•The ultraviolet loss contribution in dB/Km at any wavelength can be expressed empirically as a function of the mole fraction x of GeO₂ as

$$\alpha_{UV} = \frac{154.2x}{46.6x + 60} \times 10^{-2} \exp\left(\frac{4.63}{\lambda}\right)$$

-An empirical expression for the infrared absorption in dB/Km for $\rm GeO_2\text{-}SiO_2$ glass is

$$\alpha_{IR} = 7.81 \times 10^{11} \exp\left(\frac{-48.8}{\lambda}\right)$$

Intrinsic absorption by basic constituent atoms

•Absorption curve for a Ge0₂-doped fibre



Scattering losses

 Scattering losses in glass arise from microscopic variations in the material density, from compositional fluctuations, and for structural inhomogeneities or defects occurring during fibre manufacture.

Scattering losses

• For single-component glass the scattering loss at a wavelength λ resulting from density fluctuations can be approximated by

$$\alpha_{SCAT} = \frac{8\pi^2}{3\lambda^4} \left(n^2 - 1\right)^2 k_B T_f \beta_T$$

where

- *n* is the refractive index
- *kB* is Boltzmann's constant
- $-\beta T$ is the isothermal compressibility of the material
- *Tf* is the temperature at which the density fluctuations are frozen into the glass as it solidifies
- Another expression for the scattering loss has been derived

$$\alpha_{SCAT} = \frac{8\pi^2}{3\lambda^4} n^8 k_B T_f \beta_T$$

where

- *p* is the photoelastic coefficient

Scattering losses

• For multicomponent glasses the scattering loss is given

$$\alpha_{SCAT} = \frac{8\pi^2}{3\lambda^4} (\delta n^2)^2 \delta V$$

where the square of the mean-square refractive-index fluctuation $(\delta n^2)^2$ over a volume of δV is

$$(\delta n^2)^2 = \left(\frac{\partial n}{\partial \rho}\right)^2 (\delta \rho)^2 + \sum_{i=1}^m \left(\frac{\partial n^2}{\partial C_i}\right) (\delta C_i)$$

where

 $-\delta\rho$ is the density fluctuation

 $-\delta C_i$ is the concentration fluctuation of the *i*th glass component

Absorption

• Combining infrared, ultraviolet, and scattering losses, we get the results shown in figure below obtained for production-run graded-index multimode fibres.



• Radiative losses occur whenever an optical fibre undergoes a bend of finite radius of curvature.



- Fibres can be subject to two types of bends
 - Macroscopic bends
 - bends having radii that are large compared with the fibre diameter

Curved fiber

Random microscopic bends

bends that can arise when the fibres are incorporated into cables



Power loss from higher-order modes

- Macroscopic bends
 - This type of bends give rise to radiation losses, which are also known as *macrobending losses* or simply *bending losses*.
 - The amount of optical radiation from a bent fibre depends on the field strength at x_c and on the radius of curvature *R*.
 - The total number of modes that can be supported by a curved fibre is less than in a straight fibre and this number N_{eff} , for a curved multimode fibre of radius *a*, is

$$N_{eff} = N_{\infty} \left\{ 1 - \frac{\alpha + 2}{2\alpha\Delta} \left[\frac{2a}{R} + \left(\frac{3}{2n_2kR} \right)^{2/3} \right] \right\}$$

where

- α defines the graded index profile
- Δ is the core-cladding index difference
- n₂ is the cladding refractive index
- $k=2\pi/\lambda$ is the wave propagation constant
- and

$$N_{\infty} = \frac{\alpha}{\alpha + 2} (n_1 k a)^2 \Delta$$

is the total number of modes in a straight fibre

Example: Let us find the radius of curvature R at which the number of modes decreases by 50 percent in a graded-index fibre. For this fibre, let α =2, n₂=1.5, Δ =0.01, a=25 µm, and let the wavelength of the guided light be 1.3 µm. Solving the previous equation yields R=1.0 cm

- Random microscopic bends
 - An increase in attenuation results from microbending because the fibre curvature causes repetitive coupling of energy between the guided modes and the leaky or nonguided modes in the fibre.





One method of minimising microbending losses is by extruding a compressible jacket over the fibre



- For a multimode graded-index fibre having a core radius *a*, outer radius *b* (excluding the jacket), and index difference Δ , the microbending loss $\alpha_{\rm M}$ of a jacketed fibre is reduced from that of an unjacketed fibre by a factor

$$F(\alpha_{M}) = \left[1 + \pi \Delta^{2} \left(\frac{b}{a}\right)^{4} \frac{E_{f}}{E_{j}}\right]^{-2}$$

Core and cladding losses

 Loss for a mode of order (v,m) for a step-index waveguide

$$\alpha_{vm} = \alpha_1 \frac{P_{core}}{P} + \alpha_2 \frac{P_{clad}}{P} = \alpha_1 + (\alpha_2 - \alpha_1) \frac{P_{clad}}{P}$$

• For a graded-index fibre, the loss at distance r from core axis is

$$\alpha(r) = \alpha_1 + (\alpha_2 - \alpha_1) \frac{n^2(0) - n^2(r)}{n^2(0) - n_2^2}$$

Loss encountered by a given mode

$$\alpha_{gi} = \frac{\int_0^\infty \alpha(r) p(r) r dr}{\int_0^\infty p(r) r dr}$$

Signal distortion in optical waveguides

- Dispersion used to describe the process by which a signal propagating in a optical fibre is degraded because the various signal frequencies have different propagation velocities.
- Main causes of intra-modal dispersion or chromatic dispersion
 - Material dispersion
 - Waveguide dispersion
- Inter-modal dispersion (multimode fibres)

Pulse spreading effect

 A result of the dispersion-induced signal distortion is that a light pulse will broaden as it travels along the fibre. This pulse broadening will eventually cause a pulse to overlap with neighbouring pulses. After a certain amount of overlap has occurred, adjacent pulses can no longer be individually distinguished at the receiver and errors will occur.



Pulse spreading effect

- A measure of the information capacity of an optical waveguide is usually specified by the *bandwidth-distance product* in MHz•Km.
- For a step-index fibre the various distorsion effects tend to limit the bandwidth distance product to about 20 MHz•Km.
- In graded-index fibres the radial refractive-index profile can be carefully selected so that pulse broadening is minimised at a specific operating wavelength. This has led to bandwidth-distance products as high as 2.5 GHz•Km.
- Single-mode fibres can have capacities well in excess of this.

Group delay

- Definition: transit time required for optical power, propagating at a given mode's group velocity, to travel a given distance.
- Group delay per unit length

$$\frac{\tau_g}{L} = \frac{1}{V_g} = \frac{1}{c} \frac{d\beta}{dk} = -\frac{\lambda^2}{2\pi c} \frac{d\beta}{d\lambda}$$

$$k = 2\pi/\lambda$$

• Group velocity (Speed at which ENERGY travels)

$$V_g = c \left(\frac{d\beta}{dk}\right)^{-1} = \left(\frac{d\beta}{d\omega}\right)^{-1}$$

• Total delay difference over a distance L (simple model of two spectral components)

$$\delta \tau = \frac{d \tau_g}{d\lambda} \delta \lambda = -\frac{L}{2\pi c} \left(2\lambda \frac{d\beta}{d\lambda} + \lambda^2 \frac{d^2\beta}{d\lambda^2} \right)$$

Group delay

• In terms of frequency, expressed as

$$\delta \tau_{g} = \frac{d\tau_{g}}{d\omega} \delta \omega = -\frac{d}{d\omega} \left(\frac{L}{Vg} \right) \delta \omega = L \left(\frac{d^{2}\beta}{d\omega^{2}} \right)$$

• Pulse spreading can be approximated by rms pulse width σ_{λ}

$$\sigma_{g} = \left| \frac{d\tau_{g}}{d\lambda} \right| = \frac{L\sigma_{\lambda}}{2\pi c} \left| 2\lambda \frac{d\beta}{d\lambda} + \lambda^{2} \frac{d^{2}\beta}{d\lambda^{2}} \right|$$

• The factor D $D = \frac{1}{L} \frac{d\tau_g}{d\lambda} = \frac{d}{d\lambda} \left(\frac{1}{V_g} \right) = \frac{-2\pi c}{\lambda^2} \beta_2$ where $\beta_2 = d^2 \beta / d\omega^2$

is designated as the dispersion [ps/(nm.km)]

• Material dispersion occurs because the index of refraction varies as a function of the optical wavelength. This is exemplified in figure below for silica. As a consequence, since the group velocity V_g of a mode is a function of the index of refraction, the various spectral components of a given mode will travel at different speeds, depending on the wavelength.



 Considering a plane wave propagating in an infinitely extended dielectric medium that has a refractive index n(λ) equal to that of the fibre, the propagation constant β is thus given

$$\beta = \frac{2\pi n(\lambda)}{\lambda}$$

• The group delay resulting from material dispersion is

$$\tau_{mat} = \frac{L}{c} \left(n - \lambda \, \frac{dn}{d\lambda} \right)$$

• The pulse spread σ_{mat} for a source of spectral width σ_{λ} is found by differentiating this group delay with respect to wavelength and multiplying by σ_{λ} to yield

$$\sigma_{mat} \approx \left| \frac{d \tau_{mat}}{d \lambda} \right| \sigma_{\lambda} = \frac{\sigma_{\lambda} L}{c} \left| \lambda \frac{d^{2} n}{d \lambda^{2}} \right| = \sigma_{\lambda} L \left| D_{mat} \left(\lambda \right) \right|$$

where $D_{mat}(\lambda)$ is the material dispersion

• A plot of the material dispersion for unit length *L* and unit optical source spectral width σ_{λ} is given in figure below for the silica material



Example: Consider a typical GaAlAs LED having a spectral width of 40nm at an 800 nm output so that σ/λ = 5 percent. From Fig. 3.3 and Eq. 3-20, this produce a pulse spread of 4.4 ns/km. Not that material dispersion vanishes at 1.27 µm for pure silica.

Waveguide dispersion

- This arises in *single-mode* fibres because power exists in both cladding and core which have *slightly different* refractive indices. The effect of waveguide dispersion on pulse spreading can be approximated assuming that the refractive index of the material is independent of wavelength.
- The group delay in terms of the normalised propagation constant *b* can be defined as

$$b = 1 - \left(\frac{ua}{V}\right)^2 = \frac{\beta^2 / k^2 - n_2^2}{n_1^2 - n_2^2}$$

• For *small* values of the index difference $\Delta = (n_1 - n_2)/n_1$, the previous equation can be approximated by

$$b \approx \frac{\beta / k - n_2}{n_1 - n_2}$$

Waveguide dispersion

• Solving for β , we have

$$\beta \approx n_2 k (b\Delta + 1)$$

• The group delay τ_{wg} arising from waveguide dispersion is

$$\tau_{wg} = \frac{L}{c} \frac{d\beta}{dk} = \frac{L}{c} \left[n_2 + n_2 \Delta \frac{d(kb)}{dk} \right]$$

• Using the approximation below which is valid for small values of Δ

$$V = ka(n_1^2 - n_2^2)^{1/2} \cong kan_2\sqrt{2\Delta}$$

• The group delay can be expressed in terms of the normalised frequency *V*, yielding

$$\tau_{wg} = \frac{L}{c} \left[n_2 + n_2 \Delta \frac{d(Vb)}{dV} \right] \qquad \qquad \text{Group delay from waveguide dispersion}$$

- For single-mode fibres, waveguide dispersion is of importance and can be of the same order of magnitude as material dispersion.
- The pulse spread σ_{wg} occurring over a distribution of wavelengths σ_{λ} is obtained from the derivative of the group delay with respect to wavelength

$$\sigma_{wg} \approx \left| \frac{d \tau_{wg}}{d \lambda} \right| \sigma_{wg} = L \left| D_{wg} \left(\lambda \right) \right| \sigma_{\gamma}$$
$$= \frac{V}{\lambda} \left| \frac{d \tau_{wg}}{d \lambda} \right| \sigma_{wg} = \frac{n_2 L \Delta \sigma_{\lambda}}{c \lambda} V \frac{d^2 \left(V_b \right)}{dV^2}$$

where $D_{wg}(\lambda)$ is the waveguide dispersion.

• To see the behaviour of the waveguide dispersion, consider the expression of the factor *ua* for the lowest order mode in the normalised propagation constant. This can be approximated by

$$ua = \frac{(1+\sqrt{2})V}{1+(4+V)^{1/4}}$$

• This yields for the HE₁₁ mode

$$b(V) = 1 - \frac{\left(1 + \sqrt{2}\right)^2}{1 = \left[\left(4 + V^4\right)^{1/4}\right]^2}$$

 The figure below shows plots of the previous expression for b and its derivatives d(Vb)/dV and Vd²(Vb)/dV² as function of V



Example: From Eq. 3-26, the waveguide dispersion is

$$\sigma_{wg} = \frac{n_2 L \Delta \sigma_{\lambda}}{c \lambda} \left[V \frac{d^2 (V_b)}{d V^2} \right]$$

Let $n_2 = 1.48$ and $\Delta = 0.2$ percent. At V = 2.4, from Fig. 3.15 the expression in square brackets is 0.26. Choosing $\lambda = 1320$ nm, we have $D_{wq}(\lambda) = -1.9$ ps/(nm.km)

Chromatic dispersion



Corning SMF-28 fibre dispersion. Note that this becomes zero at 1311 nm. Near 1550 nm it is about 17 ps/(km.nm)

Figures from http://www.fiberoptics.info/articles/dispersion.htm

Effect on bandwidth



Figures and text from http://www.fiberoptics.info/articles/dispersion.htm

> "The maximum acceptable dispersion penalty is usually 2 dB, though it is possible for a system to tolerate a larger dispersion penalty if the optical attenuation is low. For the example shown in Figure [above], the maximum usable fiber length at a data rate of 3.11 Gb/s would be 85 km. At a wavelength of 1550 nm, the optical attenuation would be about 20 dB for that distance, much less than the 30 dB loss budget provided by many high-speed links. In this case, the fiber optic link would be considered dispersion-limited."

Polarisation-Mode Dispersion

- The effects of fibre birefringence on the polarisation states of an optical signal are another source of pulse broadening. This is particularly critical for high-rate long-haul transmission links that are designed to operate near the zero-dispersion wavelength of the fibre.
- *Polarisation* refers to the electric-field orientation of a light signal, which can vary significantly along the length of a fibre.
- A varying birefringence along the length of the fibre will cause each polarisation mode to travel at a slightly different velocity and the polarisation orientation will rotate with distance

Polarisation-Mode Dispersion

• The resulting difference in propagation time $\Delta \tau$ between the two orthogonal polarisation modes will result in pulse spreading. This is the *polarisation mode dispersion* (PMD)



Inter-modal distortion

- Result of different values of group delay for each mode at a single frequency
- Eliminated by *single mode* operation, but important in *multimode* fibres
- Maximum pulse spread

$$\delta T_{\rm mod} = T_{\rm max} - T_{\rm min} = \frac{n_1 \Delta L}{c}$$