Light Propagation in optical Fibres

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Introduction

- Light thought of as a wave

•Electric field component (E) can be expressed mathematically as

$$E=E_0 \cos(\omega t - kx + \varphi)$$

where

- E_0= amplitude of electric field (V/m)
- $\omega=2\pi f$=angular frequency (rad/s)
- f=optical frequency (Hz)
- t=time (s)
- $k=2\pi/\lambda$=wavenumber or propagation constant (rad/m)
- x=distance (m)
- λ=optical wavelength (m)
- φ=phase constant (rad)

Introduction

- Light thought of as a wave

•Velocity of propagation

$$v=f\lambda=c_0/n$$

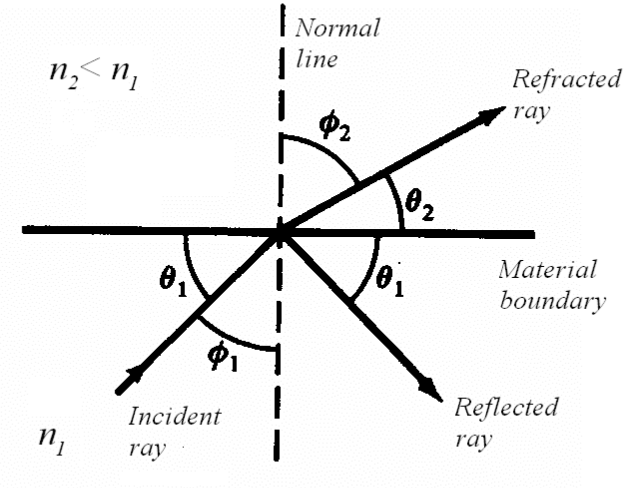
where

•$c_0 \cong 3x10^8$ m/s velocity of the light in free space

•n=refractive index of the medium in which light is propagating

Basic principles of light propagation

- "Ray Theory" (Geometrical Optics)



e.g. glass fibre cladding

e.g. glass fibre core

•Law of reflection

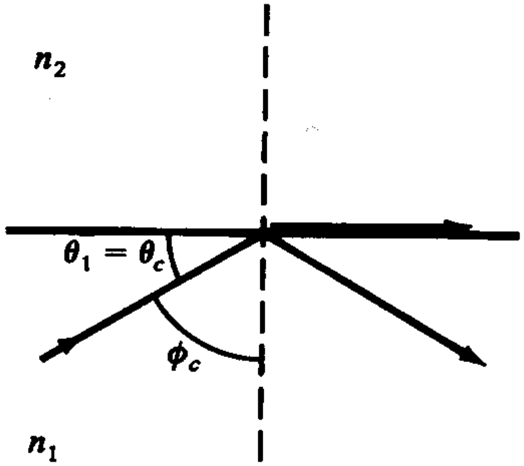
$$\varphi_I = \varphi_r$$

•Law of refraction (Snell's law)

$$n_1 \sin \varphi_1 = n_2 \sin \varphi_2$$

Basic principles of light propagation

- "Ray Theory" (Geometrical Optics)

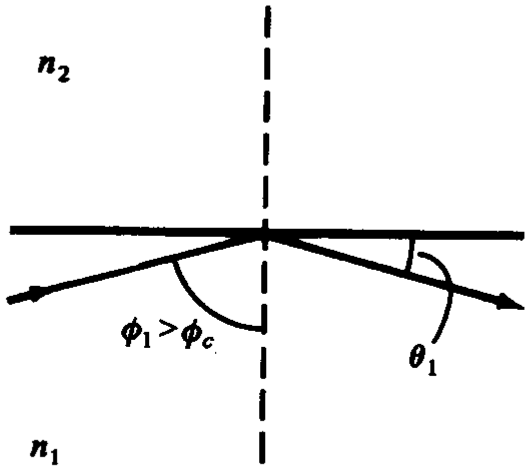


•Critical angle of incidence θ_c

$$\sin \varphi_c = n_2/n_1$$

Basic principles of light propagation

- "Ray Theory" (Geometrical Optics)



- Total Internal Reflection (T.I.R.)

Basic principles of light propagation

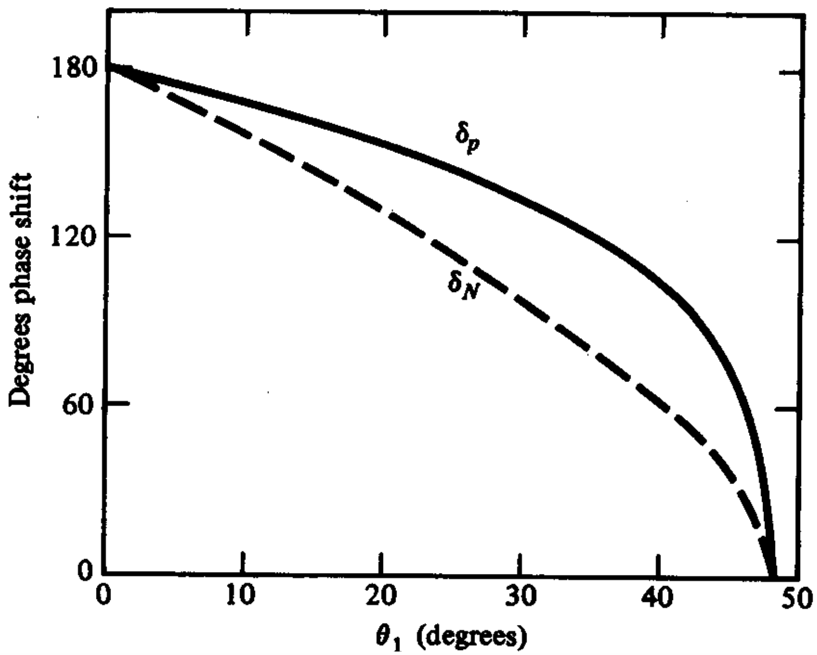
- "Ray Theory" (Geometrical Optics)

Example

Using n_1=1.50 for glass and n_2=1.00 for air, φ_c is about 52°. Any light in the glass incident on the interface at an angle φ_1 greater than 52° is totally reflected back into the glass

Basic principles of light propagation

- "Ray Theory" (Geometrical Optics)
  - When light is totally internally reflected, a phase change δ occurs in the reflected wave



Basic principles of light propagation

- "Ray Theory" (Geometrical Optics)

•This phase change depends on the angle $\theta_1 < \pi/2 - \varphi_c$ according to the relationships

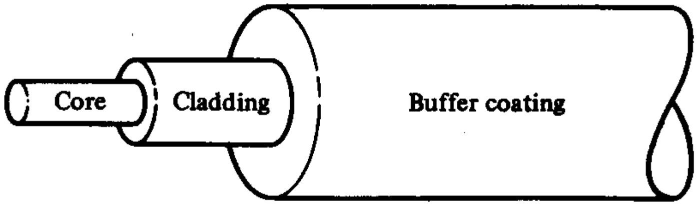
$$\tan\frac{\partial_N}{2} = \frac{\sqrt{n^2\cos^2\theta_1 - 1}}{n\sin\theta_1} \qquad \tan\frac{\partial_p}{2} = \frac{n\sqrt{n^2\cos^2\theta_1 - 1}}{\sin\theta_1}$$

δ_N and δ_P are the phase shifts of the electric-field wave components normal and parallel to the plane of incidence, respectively, and $n=n_1/n_2$

Guiding Light by T.I.R.

- Optical Fibre

An optical fibre is a dielectric waveguide that operates at optical frequencies. Typical structure of an optical fibre is shown in figure



The cylinder in the middle of the fibre is known as *core*. The core is surrounded by a solid dielectric *cladding*. The refractive index n_2 of the cladding is less than the refractive index n_1 of the core. Most fibres are encapsulated in an elastic, abrasion-resistant plastic material in order to add strength to the fibre itself.

Guiding Light by T.I.R.

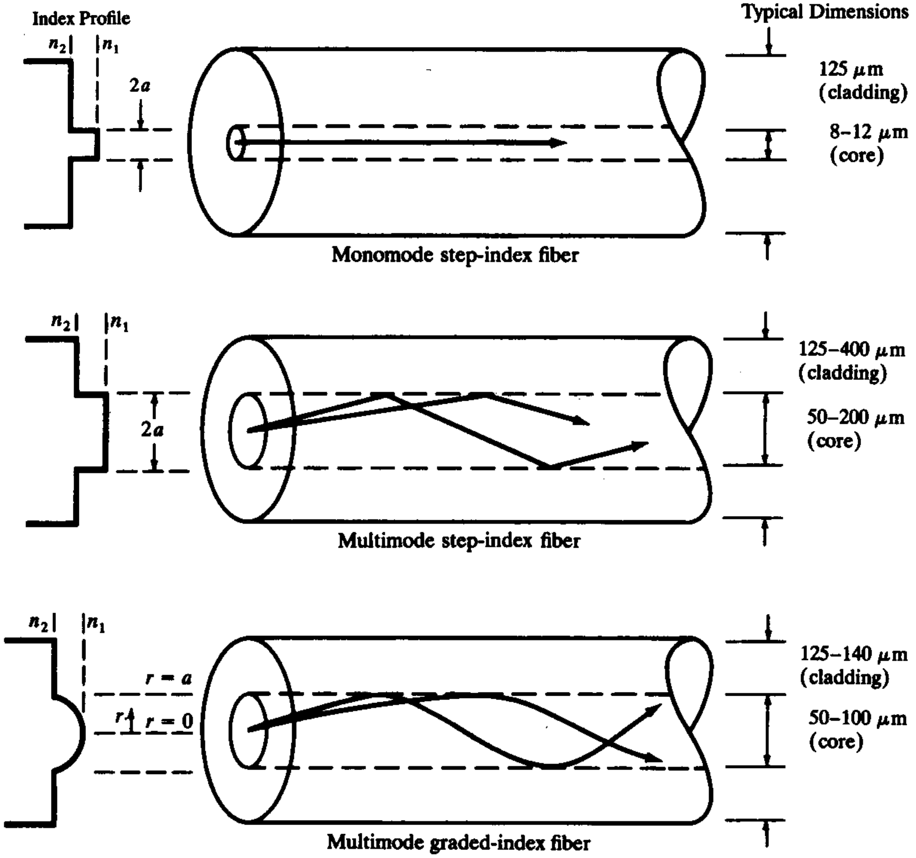
- Optical Fibre

Different type of optical fibre

•Monomode step-index fibre

•Multimode step-index fibre

•Multimode graded-index fibre



Guiding Light by T.I.R.

- Rays and Modes

•The electromagnetic light field that is guided along an optical waveguide can be represented by a superposition of bound or trapped modes. Each of these guided modes consists of a set of simple electromagnetic field configurations. For monochromatic light fields of radian frequency ω, a mode travelling in the positive z direction has a time and z dependence given by

$$e^{j(\omega t - \beta z)}$$

•The factor β is the z component of the wave propagation constant $k=2\pi/\lambda$. For guided modes, β can assume only certain discrete values.

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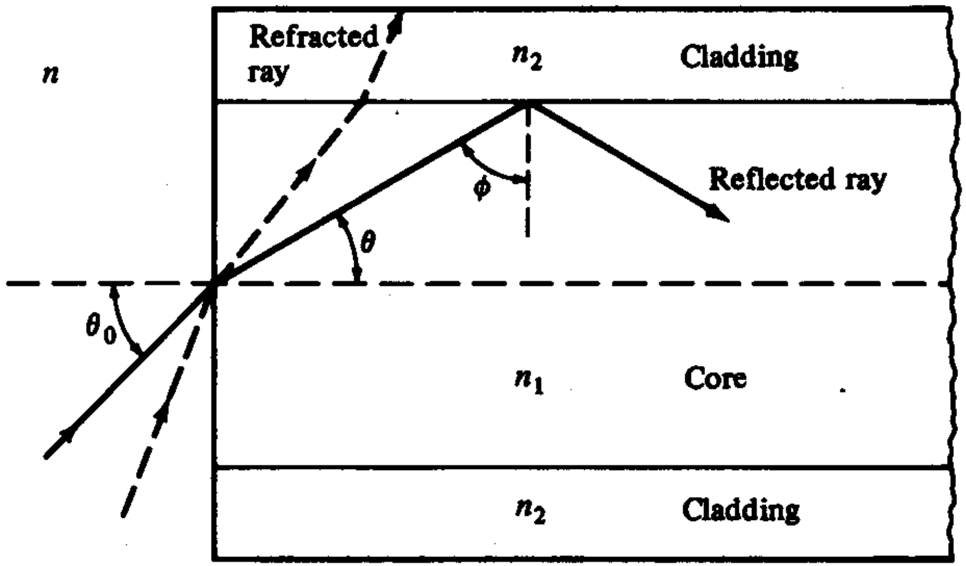
- Light propagation in optical fibre



•From Snell's law the minimum angle that supports T.I.R. is

$$\sin \varphi_{min} = n_2/n_1$$

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- Light propagation in optical fibre

•From previous equation is possible to derive the maximum entrance angle $\theta_{0,max}$

$$n\sin\theta_{0,max} = n_1 \sin\theta_c = (n_1^2 - n_2^2)^{1/2}$$

where $\theta_c = \pi/2 - \varphi_c$.

•Previous equation defines also the *Numerical Aperture*

$$NA = n\sin\theta_{0,max} \cong n_1 (2\Delta)^{1/2}$$

where $\Delta = (n_1 - n_2)/n_1$ and last relation is a good approximation if Δ is much less then 1. NA describes the gathering capability of a fibre

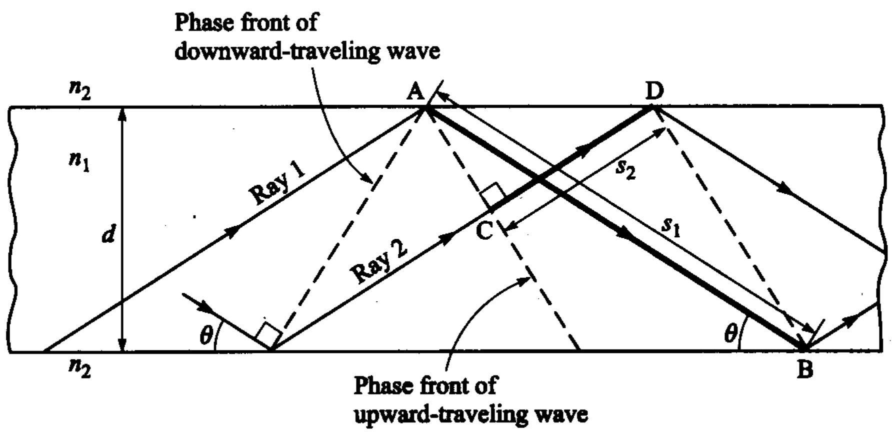
Guiding Light by T.I.R.

- Wave representation in a dielectric slab waveguide

The ray theory appears to allow rays at any angle φ greater than the critical angle φ_c to propagate along the fibre. However, when the interference effect due to the phase of the plane associated with the ray is taken into account, it is seen that only waves at certain discrete angles greater than or equal to φ_c are capable of propagating along the fibre. The condition required for a wave propagation in the dielectric slab is that all points on the same phase front of a plane wave must be in phase.

Guiding Light by T.I.R.

- Wave representation in a dielectric slab waveguide



•The phase change occurring in ray 1 travelling from point A to B minus the phase change in ray 2 between points C and D must differ by an integer multiple of 2π

$$\Delta = k_1 s = n_0 k s = n_1 2\pi s/\lambda$$

where

•k_1=the propagation constant in the medium of refractive index n_1

•$k=k_1/n_1$ is the free-space propagation constant

•s=the distance the wave has travelled in the material

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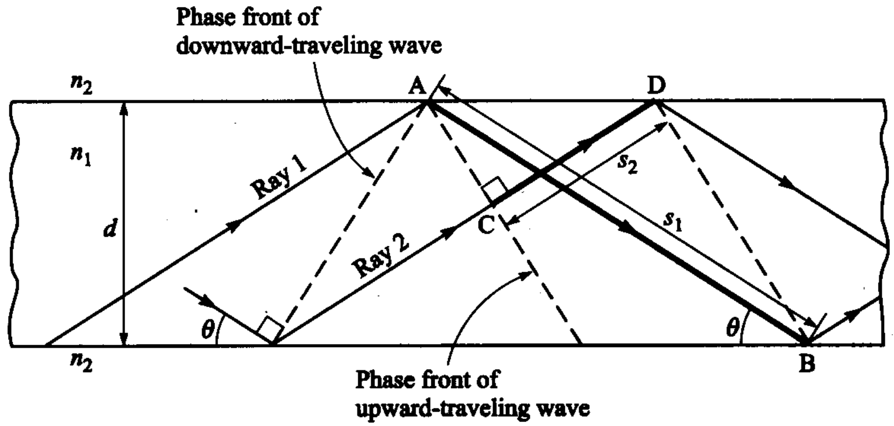
- Wave representation in a dielectric slab waveguide



•From point A to B, ray 1 travels a distance

$$s_1 = d/\sin\theta$$

and undergoes two phase changes δ at the reflection points.

•From point C to D, ray 2 doesn't incur any reflections. Note that $AD = (d/\tan\theta) - d\tan\theta$, thus the distance between C and D is

Guiding Light by T.I.R.

- Wave representation in a dielectric slab waveguide

•The requirement for wave propagation can be written as

$$\frac{2\pi n_1}{\lambda}\left(s_1 - s_2\right) + 2\partial = 2\pi m$$

where m=0,1,2,…….
Substituting the expression for s_1 and s_2

$$\frac{2\pi n_1}{\lambda}\left\{\frac{d}{\sin\theta} - \left[\frac{\left(\cos^2\theta - \sin^2\theta\right)d}{\sin\theta}\right]\right\} + 2\partial = 2\pi m$$

Which can be reduced to

$$\frac{2\pi n_1 d \sin\theta}{\lambda} + \partial = \pi m$$

Guiding Light by T.I.R.

- Wave representation in a dielectric slab waveguide

Considering only electric waves with components normal to the plane of incidence, the phase shift upon reflection is

$$\delta = -2\arctan\left[\frac{\sqrt{\cos^2\theta - \left(n_2^2/n_1^2\right)}}{\sin\theta}\right]$$

The negative sign is needed here since the wave in the medium must be a decaying and not growing wave. Substituting this expression, we obtain

$$\frac{2\pi n_1 d\sin\theta}{\lambda} - \pi m = 2\arctan\left[\frac{\sqrt{\cos^2\theta - \left(n_2^2/n_1^2\right)}}{\sin\theta}\right] \quad \text{or} \quad \tan\left(\frac{\pi n_1 d\sin\theta}{\lambda} - \frac{\pi m}{2}\right) = \left[\frac{\sqrt{n_1^2\cos^2\theta - n_2^2}}{n_1\sin\theta}\right]$$

Thus, only waves that have those angles θ will propagate in the dielectric slab waveguide

Mode theory for circular waveguides

- Overview of modes

•To attain a more detailed understanding of the optical power propagation mechanism in a fibre, it is necessary to solve Maxwell's equations subject to the boundary conditions at the interface between the core and the cladding.

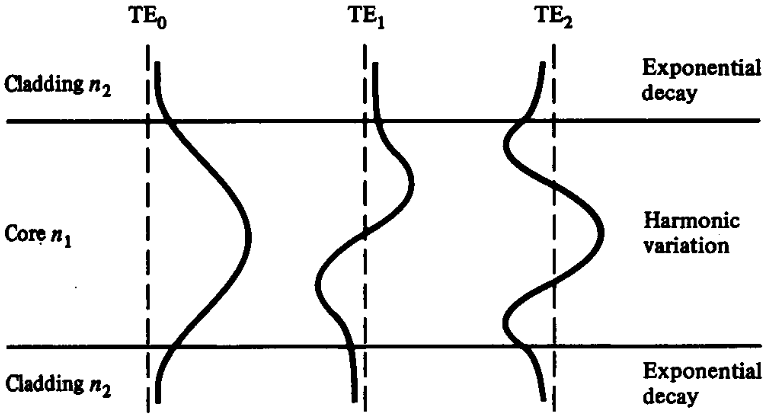
•The core-cladding boundary conditions lead to a coupling between the electric and magnetic field components. This gives rise to *hybrid modes*.

•With the assumption n_1-n_2<<1, only four field components need to be considered and their expressions become simpler. The field components are called *linearly polarised* (LP) modes and are labeled LP_{jm} where j and m are integers designating mode solutions.

Mode theory for circular waveguides

- Overview of modes

•The *order* of a mode is equal to the number of field zeros across the guide



•The fields vary harmonically in the guiding region of refractive index n_1 and decay exponentially outside of this region

•*Low order modes* are tightly concentrated near the centre of the slab

•*Higher-order modes* are distributed more towards the edges of the

Mode theory for circular waveguides

- Overview of modes

•In addition to a finite number of *guided modes*, the optical fibre waveguide has an infinite continuum of *radiation modes* that are not trapped in the core

•Some of this radiation gets trapped in the cladding causing *cladding modes* to appear

•As the core and cladding modes propagate along the fibre, mode coupling occurs between the cladding modes and the higher-order core modes

•This generally results in a loss of power from the core modes

Mode theory for circular waveguides

- Overview of modes

•In addition to bound and radiated modes, a third category of modes called *leaky modes* is present in optical fibres

•Leaky modes are only partially confined to the core region, and attenuate by continuously radiating their power out of the core as they propagate along the fibre

•This power radiation out of the waveguide results from a quantum mechanical phenomenon know as the *tunnel effect*

Mode theory for circular waveguides

- Overview of modes

•A mode remains guided as long as β satisfied the condition

$$n_2 k < \beta < n_1 k$$

where n_1 and n_2 are the refractive indices of the core and the cladding, respectively, and $k=2\pi/\lambda$

•The boundary between truly guided mode and leaky modes is defined the *cutoff* condition $\beta=n_2 k$

•As soon as β becomes smaller than $n_2 k$, power leaks out of the core into the cladding region

Mode theory for circular waveguides

- Summary of key modal concepts

•An important parameter connected with the cutoff condition is the *V number* defined by

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} NA$$

this is a dimensionless number that determines how many modes a fibre can support

•The HE_{11} mode has not cutoff and ceases to exist only when the core diameter is zero

Mode theory for circular waveguides

- Summary of key modal concepts

•The V number can also be used to express the number of modes M in a multimode fibre when V is large. An estimate of the total number of modes supported in a fibre is

$$M \approx \frac{1}{2}\left(\frac{2\pi a}{\lambda}\right)^2\left(n_1^2 - n_2^2\right) = \frac{V^2}{2}$$

•Far from cutoff the fraction of the average optical power residing in the cladding can be estimated by

$$\frac{P_{clad}}{P} \approx \frac{4}{3\sqrt{M}}$$

where P is the total optical power in the fibre

Mode theory for circular waveguides

- Summary of key modal concepts

•The number of modes that can exist in a waveguide as a function of V may be represented in terms of *normalised propagation constant b* defined by

$$b = \frac{a^2 w^2}{V^2} = \frac{(\beta / k)^2 - n_2^2}{n_1^2 - n_2^2}$$

where $w^2=\beta^2-k_2^2$, with $k_2=2\pi n_2/\lambda$

•By appropriately choosing a, n1, and n2 so that

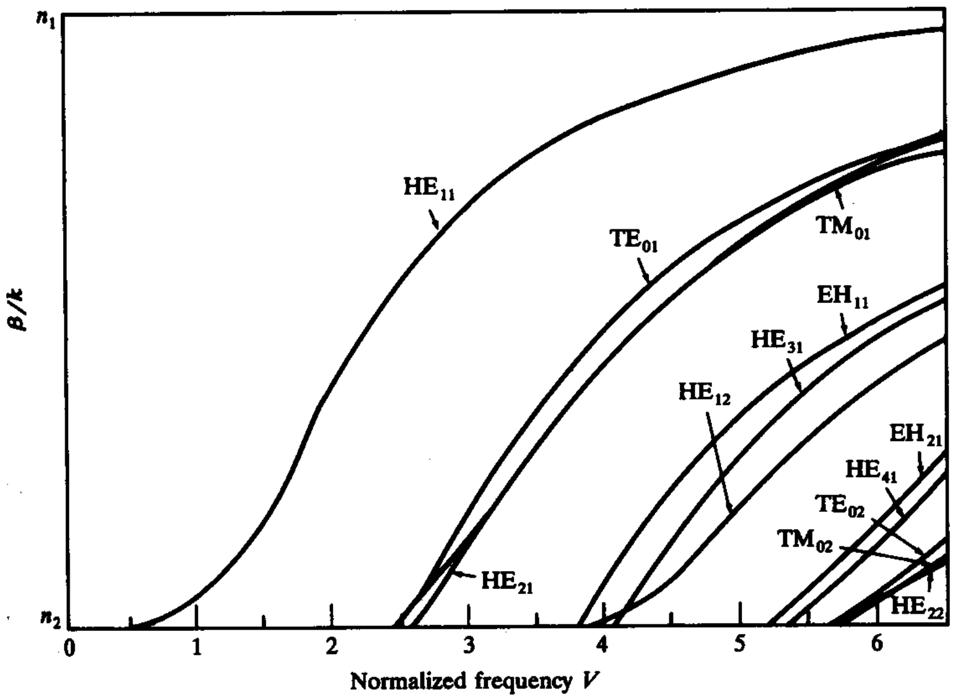
$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} \leq 2.405$$

all modes except the HE_{11} mode are cut off

Mode theory for circular waveguides

- Summary of key modal concepts

A plot of b (in term of β/k) as a function of V is shown in figure below



Mode theory for circular waveguides

- Summary of key modal concepts

Example

A step-index fibre has a normalised frequency V=26.6 at a 1300 nm wavelength. If the core radius is 25 μm, let us find the numerical aperture.

From eq.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} NA$$

or

$$NA = \frac{\lambda}{2\pi a} V = 26.6 \frac{1.3}{2\pi(25)} = 0.22$$

Mode theory for circular waveguides

- Summary of key modal concepts

  Cross-sectional views of the transverse electric field vectors for the four lowest-order modes in a step index fibre