Robust Error Square Constrained Filter Design for Systems With Non-Gaussian Noises

Fuwen Yang, Yongmin Li, and Xiaohui Liu

***Abstract*—In this letter, an error square constrained filtering problem is considered for systems with both non-Gaussian noises and polytopic uncertainty. A novel filter is developed to estimate the systems states based on the current observation and known deterministic input signals. A free parameter is introduced in the filter to handle the uncertain input matrix in the known deterministic input term. In addition, unlike the existing variance constrained filters, which are constructed by the previous observation, the filter is formed from the current observation. A time-varying linear matrix inequality (LMI) approach is used to derive an upper bound of the state estimation error square. The optimal bound is obtained by solving a convex optimization problem via semi-definite programming (SDP) approach. Simulation results are provided to demonstrate the effectiveness of the proposed method.**

***Index Terms*—Current observation, error square constrained filtering, known deterministic input, non-Gaussian noise, polytopic uncertainty.**

I. Introduction

KALMAN filter has been widely applied in many engineering and information systems, for instance, target tracking, image processing, signal processing, communication, and control engineering [1]. However, filtering performance may deteriorate by use of standard Kalman filter when the underlying systems contain parameter uncertainties and non-Gaussian noises due to unmodeled dynamics, parameter variations, model reduction, linearization, and external severe environment [18]. There are essentially two approaches to cope with parameter uncertainties and/or non-Gaussian noises. One is robust filtering and the other is H_∞ filtering. H_∞ filtering method provides an energy bounded gain from the noise inputs to the estimation error without the need for knowledge of noise statistics [17]. In this filtering, process and measurement noises are assumed to be arbitrary rather than Gaussian processes. It has been proven that H_∞ filtering is less sensitive to parameter uncertainties and non-Gaussian noises, but its design is too conservative and there is no provision to ensure that the variance of the state estimation error lies within acceptable bounds [17]. Robust filtering has attempted to constrain the variance in spite of large parameter uncertainties [4], [5], [10], [11]. There has been a number of literature to address the robust filtering problems with variance constrained [13], [14], [16]. The robust variance constrained filtering problems were considered for linear systems with norm-bounded parameter uncertainties [16]. The filter was obtained by solving two Riccati-like equations, where a scaling parameter is searched to find a feasible solution [3]. In order to avoid the scaling parameter search, an LMI approach has been applied to solve for linear systems with both norm-bounded parameter uncertainties and polytopic uncertainties [3], [15]. Recently, this problem has been extensively studied. For example, the robust variance constrained filtering problem for uncertain systems with multiplicative noises has been considered in [12] and [18]; the robust variance constrained filtering problem for uncertain systems with stochastic nonlinearities has been studied in [19]; and the robust variance constrained filtering problem for uncertain systems with random sensor delays has been solved in [20]. So far, to the best of our knowledge, it is always assumed that the process noises and measurement noises are Gaussian white ones in the existing literature about robust variance constrained filtering. However, in practical applications, the process noises and measurement noises may be non-Gaussian [6]. This motivates us to investigate the robust filter design problems for non-Gaussian noises. Since the disturbance and measurement noises are treated as hard constraints and not random variables, we call this filter as an error square constrained filter. Moreover, we will also investigate the filtering problem for uncertain systems containing known deterministic input [7]. In deterministic systems, the existence of a known deterministic input is of no significance for the filtering problem, as the filter can be designed to cancel the effect on the estimation error. When there exist parameter uncertainties in systems, the known deterministic input will produce an unknown bias in filtering error [7]. In order to avoid the design difficulty, most of the existing works in this area assume that the system is driven only by noise processes without the presence of a known deterministic input [3]–[5], [10]–[12], [15], [16], [18]–[20].

In this letter, we will present several novel techniques to tackle these two problems. We combine them as a robust error square constrained filtering problem for uncertain system with known deterministic input and non-Gaussian noises. In particular, a novel filter is proposed based on current observation and known deterministic input signals. A free parameter is introduced in the filter to handle the uncertain input matrix in the known deterministic input term. In addition, unlike the existing variance constrained filters, which are constructed by the previous observation, the filter is formed from the current observation. An upper bound of the state estimation error square is derived from the system equation and the filter equation, which is an inequality containing non-Gaussian process and measurement noises constraints. S-procedure and Schur complement techniques are employed to combine all

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inequalities into one time-varying LMI. The convex combination approach is applied to handle the polytopic uncertainties in the LMI. Finally, the filtering problem is transferred into a convex optimization problem, which is easily solved via the SDP approach.

II. Problem Formulation

Consider the following discrete-time polytopic uncertain system:

$$x_{k+1} = A_k(\alpha)x_k + F_k(\alpha)u_k + B_k(\alpha)w_k \tag{1}$$

$$y_k = C_k x_k + D_k v_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^l$ is the known deterministic input, $y_k \in \mathbb{R}^m$ is the measurement output, $w_k \in \mathbb{R}^r$ is the process noise, and $v_k \in \mathbb{R}^p$ is the measurement noise.

w_k and v_k are non-Gaussian noise signals at time step k, which are assumed to satisfy the following constraints:

$$w_k^T Q_k^{-1} w_k \le 1 \tag{3}$$

$$v_k^T R_k^{-1} v_k \le 1 \tag{4}$$

where $Q_k = Q_k^T > 0$ and $R_k = R_k^T > 0$ are known matrices with compatible dimensions. The initial state x_0 is also assumed to satisfy a constraint

$$(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T \le P_0 \tag{5}$$

where $\hat{x}_0$ is an estimate of x_0 which is assumed to be given, and $P_0 = P_0^T > 0$ is a known matrix. The matrices $A_k(\alpha)$, $B_k(\alpha)$, and $F_k(\alpha)$ are unknown time-varying parameters with appropriate dimensions. We assume that $\big(A_k(\alpha), B_k(\alpha), F_k(\alpha)\big) \in \Omega$, where Ω is a convex polyhedral set described by K vertices

$$\Omega = \left\{ (A_k(\alpha), B_k(\alpha), F_k(\alpha)) = \sum_{i=1}^{K} \alpha_i \left(A_k^{(i)}, B_k^{(i)}, F_k^{(i)}\right), \sum_{i=1}^{K} \alpha_i = 1, \alpha_i \ge 0 \right\} \tag{6}$$

where $\left(A_k^{(i)}, B_k^{(i)}, F_k^{(i)}\right)$ are known for all $i = 1, 2, \ldots, K$.

In this letter, a novel filter based on the current observation is developed for the uncertain system (1) and (2). The filter is described in the following form:

$$\hat{x}_{k+1} = G_k \hat{x}_k + H_k u_k + L_k y_{k+1} \tag{7}$$

where $\hat{x}_k \in \mathbb{R}^n$ is the state estimate of x_k, G_k, H_k, and L_k are the filter parameters to be determined.

Our objective is first to design the filter (7) such that an upper bound for the estimation error square is guaranteed for all unknown matrices $(A_k(\alpha), B_k(\alpha), F_k(\alpha)) \in \Omega$, and then minimize such a bound in the sense of the matrix trace, that is, to find the filter (7) and a sequence of positive-definite matrices P_{k+1} $(0 < k \le N-1)$ such that

$$\min_{P_{k+1}, G_k, H_k, L_k} \quad \operatorname{trace}(P_{k+1}) \tag{8}$$

$$\text{subject to} \quad (x_{k+1} - \hat{x}_{k+1})(x_{k+1} - \hat{x}_{k+1})^T \le P_{k+1} \tag{9}$$

and the constraints (3) and (4). This problem will be referred to as a robust error square constrained filter design problem.

III. Robust Error Square Constrained Filter Design

In this section, a robust error square constrained filter will be designed for discrete-time polytopic uncertain systems subject to any non-Gaussian process noise and measurement noise satisfying (3) and (4).

We first consider the system (1) and (2) with the known parameters $(A_k, B_k, C_k, D_k, F_k)$, which is defined as a deterministic system.

Theorem 1: For the deterministic system (1) and (2), the solution to the optimization problem (8) subject to (9) and the constraints (3) and (4) is obtained by solving the convex optimization problem

$$\min_{P_{k+1}>0, G_k, L_k, \tau_1 \ge 0, \tau_2 \ge 0, \tau_3 \ge 0} \quad \operatorname{trace}(P_{k+1}) \tag{10}$$

$$\text{subject to} \begin{bmatrix} P_{k+1} & \Pi(\hat{x}_k, u_k) \\ \Pi(\hat{x}_k, u_k)^T & Q_\tau \end{bmatrix} \ge 0 \tag{11}$$

where $Q_\tau = \operatorname{diag}(1 - \tau_1 - \tau_2 - \tau_3, \tau_1 I, \tau_2 Q_k^{-1}, \tau_3 R_{k+1}^{-1})$ and $\Pi(\hat{x}_k, u_k)$ is defined in (12) at the bottom of the page. Moreover, the filter is given by

$$\hat{x}_{k+1} = G_k \hat{x}_k + F_k u_k + L_k y_{k+1}. \tag{13}$$

Proof: From (1) and (2) and (7), the estimation error $x_{k+1} - \hat{x}_{k+1}$ is written as

$$\begin{aligned} x_{k+1} - \hat{x}_{k+1} &= A_k x_k + F_k u_k + B_k w_k - G_k \hat{x}_k - H_k u_k - L_k y_{k+1} \\ &= (I - L_k C_{k+1}) A_k x_k - G_k \hat{x}_k \\ &\quad + [(I - L_k C_{k+1}) F_k - H_k] u_k \\ &\quad + (I - L_k C_{k+1}) B_k w_k - L_k D_{k+1} v_{k+1}. \end{aligned} \tag{14}$$

If $(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T \le P_k$, then there exists a z with $\|z\| \le 1$ such that

$$x_k = \hat{x}_k + E_k z \tag{15}$$

where E_k is a factorization of $P_k = E_k E_k^T$. Substituting (15) into (14) yields

$$\begin{aligned} x_{k+1} - \hat{x}_{k+1} = &(I - L_k C_{k+1}) A_k \hat{x}_k - G_k \hat{x}_k \\ &+ [(I - L_k C_{k+1}) F_k - H_k] u_k \\ &+ (I - L_k C_{k+1}) A_k E_k z \\ &+ (I - L_k C_{k+1}) B_k w_k - L_k D_{k+1} v_{k+1}. \end{aligned} \tag{16}$$

$$\Pi(\hat{x}_k, u_k) = \left[(I - L_k C_{k+1}) A_k \hat{x}_k - G_k \hat{x}_k - L_k C_{k+1} F_k u_k \;\; (I - L_k C_{k+1}) A_k E_k \quad (I - L_k C_{k+1}) B_k \quad -L_k D_{k+1} \right] \tag{12}$$

Since it is assumed that the system (1) and (2) is known, we can choose

$$H_k = F_k. \tag{17}$$

Then, (16) is simplified as

$$\begin{aligned} x_{k+1} - \hat{x}_{k+1} = &(I - L_k C_{k+1}) A_k \hat{x}_k \\ &- G_k \hat{x}_k - L_k C_{k+1} F_k u_k + (I - L_k C_{k+1}) \\ &\cdot A_k E_k z + (I - L_k C_{k+1}) B_k w_k \\ &- L_k D_{k+1} v_{k+1}. \end{aligned} \tag{18}$$

Defining

$$\eta = [1 \quad z^T \quad w_k^T \quad v_{k+1}^T]^T \tag{19}$$

we can rewrite (18) as follows:

$$x_{k+1} - \hat{x}_{k+1} = \Pi(\hat{x}_k, u_k)\eta \tag{20}$$

where $\Pi(\hat{x}_k, u_k)$ is defined in (12). Hence, $(x_{k+1} - \hat{x}_{k+1})(x_{k+1} - \hat{x}_{k+1})^T \leq P_{k+1}$ can be written as

$$P_{k+1} - \Pi(\hat{x}_k, u_k)\eta\eta^T \Pi(\hat{x}_k, u_k)^T \geq 0. \tag{21}$$

By using Schur complements, (21) is transferred as

$$\begin{bmatrix} P_{k+1} & \Pi(\hat{x}_k, u_k)\eta \\ \eta^T \Pi(\hat{x}_k, u_k)^T & 1 \end{bmatrix} \geq 0. \tag{22}$$

Performing the congruence transformation $\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$ to (21) yields

$$\begin{bmatrix} 1 & \eta^T \Pi(\hat{x}_k, u_k)^T \\ \Pi(\hat{x}_k, u_k)\eta & P_{k+1} \end{bmatrix} \geq 0 \tag{23}$$

which is equivalent to

$$1 - \eta^T \Pi(\hat{x}_k, u_k)^T P_{k+1}^{-1} \Pi(\hat{x}_k, u_k)\eta \geq 0 \tag{24}$$

by using Schur complements. By the definition of η in (19), (24) is written as

$$\eta^T \mathrm{diag}(1, 0, 0, 0)\eta - \eta^T \Pi(\hat{x}_k, u_k)^T P_{k+1}^{-1} \Pi(\hat{x}_k, u_k)\eta \geq 0. \tag{25}$$

Now $\|z\| \leq 1$, $w_k^T Q_k^{-1} w_k \leq 1$, and $v_{k+1}^T R_{k+1}^{-1} v_{k+1} \leq 1$ are also written as

$$\eta^T \mathrm{diag}(1, -I, 0, 0)\eta \geq 0 \tag{26}$$

$$\eta^T \mathrm{diag}\left(1, 0, -Q_k^{-1}, 0\right)\eta \geq 0 \tag{27}$$

$$\eta^T \mathrm{diag}\left(1, 0, 0, -R_{k+1}^{-1}\right)\eta \geq 0. \tag{28}$$

By using S-procedure, the sufficient condition such that the inequalities (26)–(28) imply (25) to hold is that there exist non-negative scalars τ_1, τ_2, and τ_3 such that

$$\begin{aligned} &\mathrm{diag}(1, 0, 0, 0) - \Pi(\hat{x}_k, u_k)^T P_{k+1}^{-1} \Pi(\hat{x}_k, u_k) \\ &\quad - \tau_1 \mathrm{diag}(1, -I, 0, 0) \\ &\quad - \tau_2 \mathrm{diag}\left(1, 0, -Q_k^{-1}, 0\right) - \tau_3 \mathrm{diag}\left(1, 0, 0, -R_{k+1}^{-1}\right) \geq 0. \end{aligned} \tag{29}$$

Equation (29) is written in the following compact form:

$$\begin{aligned} &\mathrm{diag}\left(1 - \tau_1 - \tau_2 - \tau_3, \tau_1 I, \tau_2 Q_k^{-1}, \tau_3 R_{k+1}^{-1}\right) \\ &\quad - \Pi(\hat{x}_k, u_k))^T P_{k+1}^{-1} \Pi(\hat{x}_k, u_k) \geq 0. \end{aligned} \tag{30}$$

By using Schur complements, (30) is equivalent to (11). ■

For polytopic uncertain systems, we cannot employ Theorem 1 to obtain an optimized upper bound of the state estimation error square and the corresponding filter. Now we apply the convex combination approach proposed by [9], [15] to cope with polytopic uncertain systems.

Theorem 2: For the polytopic uncertain system (1) and (2) whose parameters reside in polytope Ω(6) with given vertices $A_k^{(i)}$, $B_k^{(i)}$, and $F_k^{(i)}$ $(i = 1, 2, \ldots, K)$, the solution to the optimization problem (8) subject to (9) and the constraints (3) and (4) is obtained by solving the convex optimization problem

$$\min_{P_{k+1}>0, G_k, H_k, L_k, \tau_1 \geq 0, \tau_2 \geq 0, \tau_3 \geq 0} \mathrm{trace}(P_{k+1}) \tag{31}$$

$$\text{subject to} \begin{bmatrix} P_{k+1} & \Pi^{(i)}(\hat{x}_k, u_k) \\ \Pi^{(i)}(\hat{x}_k, u_k)^T & Q_\tau \end{bmatrix} \geq 0 \tag{32}$$

where

$$\begin{aligned} \Pi^{(i)}(\hat{x}_k, u_k) = \big[&\Pi_1^{(i)}(\hat{x}_k, u_k) \quad (I - L_k C_{k+1}) A_k^{(i)} E_k \\ &(I - L_k C_{k+1}) B_k^{(i)} \quad -L_k D_{k+1} \big] \end{aligned} \tag{33}$$

and

$$\begin{aligned} \Pi_1^{(i)}(\hat{x}_k, u_k) = &(I - L_k C_{k+1}) A_k^{(i)} \hat{x}_k - G_k \hat{x}_k - H_k u_k \\ &+ (I - L_k C_{k+1}) F_k^{(i)} u_k \end{aligned} \tag{34}$$

for all $i \in \{1, 2, \ldots, K\}$. Moreover, the filter is given by

$$\hat{x}_{k+1} = G_k \hat{x}_k + H_k u_k + L_k y_{k+1} \tag{35}$$

where $A_k^{(i)}$, $B_k^{(i)}$, and $F_k^{(i)}$ are the matrices in (6) at the ith vertex of the polytope.

Proof: The proof can follow the one in Theorem 1. We give the sketch of the proof here. It is noted that $F_k(\alpha)$ is an uncertain matrix. Therefore, we cannot use (17) to design the filter parameter H_k, and $x_{k+1} - \hat{x}_{k+1}$ is written as

$$x_{k+1} - \hat{x}_{k+1} = \Pi(\hat{x}_k, u_k)\eta \tag{36}$$

where $\Pi(\hat{x}_k, u_k)$ is defined in (33) and η defined in (19). The condition $(x_{k+1} - \hat{x}_{k+1})(x_{k+1} - \hat{x}_{k+1})^T \leq P_{k+1}$ subject to the constraints (3) and (4) is equivalent to

$$\begin{bmatrix} P_{k+1} & \Pi(\hat{x}_k, u_k) \\ \Pi(\hat{x}_k, u_k)^T & Q_\tau \end{bmatrix} \geq 0. \tag{37}$$

Now we prove the equivalence between (37) and (33) for all $i \in \{1, 2, \ldots, K\}$. Necessity is straightforward since if (37) is satisfied for all the polytope, it must be the case at all the vertices. Sufficiency is deduced from

$$\begin{aligned} &\begin{bmatrix} P_{k+1} & \Pi(\hat{x}_k, u_k) \\ \Pi(\hat{x}_k, u_k)^T & Q_\tau \end{bmatrix} \\ &\quad = \sum_{i=1}^{K} \alpha_i \begin{bmatrix} P_{k+1} & \Pi^{(i)}(\hat{x}_k, u_k) \\ \Pi^{(i)}(\hat{x}_k, u_k)^T & Q_\tau \end{bmatrix} \\ &\quad \geq 0 \end{aligned} \tag{38}$$

by noting that $\alpha_i \geq 0$ and $\sum_{i=1}^{K} \alpha_i = 1$. $\Pi^{(i)}(\hat{x}_k, u_k)$ is defined in (33). ■

Remark 1: We can see from Theorem 2 that the inequalities (32) are linear to the variables P_{k+1}, G_k, H_k, and L_k, τ_1, τ_2, and τ_3, Hence, the optimization problems (31) can be solved by the existing SDP via the interior-point approach.

Remark 2: The trace of P_{k+1} is optimized at each time step in an effort to find the smallest error for the state estimate. Other measures can also be introduced, for example, determinant [5].

IV. Illustrative Example

Consider an uncertain system

$$x_{k+1} = \begin{bmatrix} 0 & -0.95 \\ 0.9+\alpha & 0.8 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u_k + \begin{bmatrix} 10 \\ 2 \end{bmatrix} w_k$$
$$y_k = [0.1 \quad 0.5]\, x_k + v_k.$$

Due to modeling errors, α is unknown but it belongs to the known interval $[\alpha_{min}, \alpha_{max}]$, where $\alpha_{min} = -0.03$ and $\alpha_{max} = 0.03$.

In the simulation, w_k and v_k are considered as outliers, which appear once after every five recursive steps and which magnitudes are 0.5. The input is set as $u_k = 10$. The initial state and state estimate are assumed as $x_0 = [5 \ -5]^T$ and $\hat{x}_0 = [0 \ 0]^T$, respectively. The initial condition is assumed as $P_0 = \begin{bmatrix} 100 & 0 \\ 0 & 50 \end{bmatrix}$. For all k, $Q_k = 0.25$ and $R_k = 0.25$.

The simulation results are obtained by solving the convex optimization problem (31) and (32) in Theorem 2 under Matlab 6.5 with YALMIP 3.0 and SeDuMi 1.1 [8]. The simulation results show that the actual error square of the states stay below their upper bounds. Therefore, the proposed design method provides an expected error square constraint. However, due to the non-Gaussian noises, the upper bounds seem too conservative.

V. Conclusions

In this letter, an error square constrained filtering problem has been considered for discrete-time systems with polytopic uncertainty and non-Gaussian noises. The proposed filter has been constructed from the current observation and known deterministic input signals. The time-varying LMI approach has been applied to derive an upper bound of the state estimation error square which is optimized by solving a convex optimization problem via SDP approach. An illustrative example has demonstrated the feasibility of the proposed filtering methods. The proposed filtering algorithm is similar to recursive Kalman filtering one.

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