www.ietdl.org

Published in IET Signal Processing
Received on 6th October 2009
Revised on 12th June 2010
doi: 10.1049/iet-spr.2009.0244

ISSN 1751-9675

Robust set-membership filtering for systems with missing measurement: a linear matrix inequality approach

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Abstract: This study addresses the robust set-membership finite-horizon filtering problem for a class of discrete time-varying systems with missing measurement and polytopic uncertainties in the presence of unknown-but-bounded process and measurement noises. A robust set-membership filter is developed and a recursive algorithm is derived for computing the state estimate ellipsoid that is guaranteed to contain the true state. An optimal possible estimate set is computed recursively by solving the semi-definite programming problem. Simulation results are provided to demonstrate the effectiveness of the proposed method.

1 Introduction

Filtering plays an important role in signal processing [1]. It is well-known that the Kalman filter requires the process and measurement noises to be white Gaussian processes [2]. However, the Kalman filter may lead to poor performance for non-Gaussian noises [3]. Recently, the H_∞ filtering method has been proposed, which provides an energy bounded gain for the worst-case estimation error without the need for knowledge of noise statistics [4]. In this filtering, process and measurement noises are assumed to be arbitrary rather than Gaussian processes. However, there is no provision in H_∞ filtering to ensure that the variance of the state estimation error lies within acceptable bounds [5]. In this respect, it is natural to consider process and measurement noises as unknown-but-bounded which belong to given sets in appropriate vector spaces [6, 7]. All possible state estimates can be characterised by the set of state estimates consistent with both the measurements received and the constraints on the unknown process and measurement noises whose norms are less than the prescribed scalars, and the true state is contained in this set of state estimates. The actual estimate thus is a set in state space rather than a single vector. This estimation problem has been referred to as a set-membership (set-valued) filtering problem [6, 8–12].

The set-membership filtering problem was first considered by Witsenhausen [13]. The set of all possible values of the states compatible with the measurement of outputs is completely characterised by their support functions. An ellipsoidal approximation algorithm was provided by Schweppe [7]. In this algorithm, the measurements are used to calculate recursively a bounding ellipsoid to the set of possible states, under the assumption that the sets containing the initial condition and the process and measurement noises are, or can be approximated, by ellipsoids. The solution to a set-membership filtering problem with the instantaneous constraints was determined by using the results derived for an energy constraint in [8]. The resulting estimator is similar to that proposed by Schweppe [7]; however it has an important advantage that the gain matrix does not depend on the particular output measurements and is therefore precomputable. Recently, many researchers have attempted to deal with the set-membership filtering problems for various uncertain systems. For example, a convex optimisation approach has been applied to the case of norm-bounded uncertainty in the system matrices to provide a set of state estimates in [14]. A combinational ellipsoidal constraint of the uncertain system matrix and uncertain process noise was introduced for set-membership filtering in [15]. A recursive scheme for constructing an ellipsoidal state estimation set of all states consistent with the measured output and the given noise and unstructured uncertainty described by a sum quadratic constraint was presented in [16–18]. In the existing literature concerning set-membership filtering techniques, it is implicitly assumed that the measurements always contain consecutive useful signals; see for example [19–21]. In many real-world applications, however, the measurements are not consecutive but contain missing measurements. For example, the missing measurements exist in signal shading and blocking, intermittent sensor failure, network congestion, accidental data packet loss of network communication and so on. This motivates us to investigate the set-membership filtering problem with missing measurement. It is important to note that, to our best knowledge, no paper has been published on this specific topic.

There have been two ways to model the missing measurement phenomena, that is, using stochastic binary switching sequence and using deterministic binary switching sequence. The deterministic binary switching sequence is a switching sequence, which is known a priori, whereas the stochastic binary switching sequence is a sequence that cannot be predicted. The stochastic binary switching sequence is specified by a conditional probability distribution and enters into system measurement [22]. It can be viewed as a Bernoulli distributed white sequence. The model was employed in [23, 24] to study the robust filter design problem with error variance constraints. Another way is to model the missing measurement as a deterministic binary switching sequence. An incompleteness matrix has been introduced to quantify the missing data in [25, 26]. The robust filtering problem with missing data has been investigated in terms of a recursive state estimator. In this paper, we model the missing measurement by a deterministic binary switching sequence taking on the values of 0 and 1. The systems under consideration have polytopic uncertainty and unknown-but-bounded process and measurement noises. Our aim is to design a robust set-membership filter for polytopic uncertain systems with missing measurement in the presence of unknown-but-bounded process and measurement noises. We adopt the S-procedure technique [27] to determine a state estimation ellipsoid that is a set of states compatible with the incomplete (missing) measurement, the constraints on unknown-but-bounded process and measurement noises and polytopic uncertainty. A recursive algorithm is developed for computing the ellipsoid that is guaranteed to contain the true state. In each step, the ellipsoid is minimised in some sense by solving a semi-definite programming problem. This ellipsoid is an optimal possible estimate set, which can be calculated recursively in real time.

The remainder of this paper is organised as follows. The robust set-membership filter design problem is formulated in Section 2 for discrete time-varying polytopic uncertain systems with missing measurement. A novel set-membership filtering algorithm for computing the state estimation ellipsoid is developed in Section 3. Section 4 provides an illustrative example to demonstrate the effectiveness of our algorithm. Conclusions and future directions are described in Section 5.

Notation: The notation $\boldsymbol{X} \geq \boldsymbol{Y}$ (respectively, $\boldsymbol{X} > \boldsymbol{Y}$) where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are symmetric matrices, means that $\boldsymbol{X} - \boldsymbol{Y}$ is positive semi-definite (respectively, positive definite). The superscript T stands for matrix transposition. The notation trace $(\boldsymbol{P})$ denotes the trace of $\boldsymbol{P}$.

2 Problem formulation

Consider the following discrete time-varying polytopic uncertain system [28]

$$\boldsymbol{x}_{k+1} = A_k(\alpha)\boldsymbol{x}_k + F_k(\alpha)\boldsymbol{u}_k + B_k(\alpha)\boldsymbol{w}_k \tag{1}$$

where $\boldsymbol{x}_k \in \mathbb{R}^n$ is the system state, $\boldsymbol{u}_k \in \mathbb{R}^l$ is the known deterministic input, and $\boldsymbol{w}_k \in \mathbb{R}^r$ is the process noise.

The matrices $\boldsymbol{A}_k(\alpha)$, $\boldsymbol{B}_k(\alpha)$ and $\boldsymbol{F}_k(\alpha)$ are unknown time-varying matrices with appropriate dimensions, where α is an uncertain parameter. We assume that $(\boldsymbol{A}_k(\alpha), \boldsymbol{B}_k(\alpha), \boldsymbol{F}_k(\alpha)) \in \Omega_1$, where Ω_1 is a convex polyhedral set described by N vertices

$$\begin{aligned}\Omega_1 = \Big\{&(\boldsymbol{A}_k(\alpha), \boldsymbol{B}_k(\alpha), \boldsymbol{F}_k(\alpha))\\ &= \sum_{i=1}^{N} \alpha_i(\boldsymbol{A}_k^{(i)}, \boldsymbol{B}_k^{(i)}, \boldsymbol{F}_k^{(i)}), \ \sum_{i=1}^{N} \alpha_i = 1, \ \alpha_i \geq 0\Big\}\end{aligned} \tag{2}$$

where $(\boldsymbol{A}_k^{(i)}, \boldsymbol{B}_k^{(i)}, \boldsymbol{F}_k^{(i)})$ are known for all $i = 1, 2, \ldots, N$.

The measurements, which may appear as missing data occasionally, are described by

$$y_k = \gamma_k(\boldsymbol{C}_k(\alpha)\boldsymbol{x}_k + \boldsymbol{D}_k(\alpha)\boldsymbol{v}_k) \tag{3}$$

where $y_k \in \mathbb{R}^m$ is the measurement output; $\boldsymbol{v}_k \in \mathbb{R}^p$ is the measurement noise. $\gamma_k \in \mathbb{R}$ takes the values of 0 and 1, that is, the measurement data y_k is available if $\gamma_k = 1$ and y_k is missing if $\gamma_k = 0$; The matrices $\boldsymbol{C}_k(\alpha)$ and $\boldsymbol{D}_k(\alpha)$ are unknown time-varying matrices with appropriate dimensions. We assume that $(\boldsymbol{C}_k(\alpha), \boldsymbol{D}_k(\alpha)) \in \Omega_2$, where Ω_2 is a convex polyhedral set described by N vertices

$$\Omega_2 = \left\{(\boldsymbol{C}_k(\alpha), \boldsymbol{D}_k(\alpha)) = \sum_{i=1}^{N} \alpha_i(\boldsymbol{C}_k^{(i)}, \boldsymbol{D}_k^{(i)}), \ \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0\right\} \tag{4}$$

where $(\boldsymbol{C}_k^{(i)}, \boldsymbol{D}_k^{(i)})$ are known for all $i = 1, 2, \ldots, N$.

Remark 1: As compared with the norm-bounded uncertainty in [3, 5, 14], the polytopic uncertainty considered in this paper is more flexible. Polytopic uncertainty is probably the most general way of capturing the structured uncertainty that may affect the system parameters. It includes the well-known interval parametric uncertainty [28].

Remark 2: This is the first to consider the missing measurement problem in set-membership filtering. The missing measurement is described as a binary switching sequence which is viewed as a Bernoulli distributed white sequence taking on the values of 0 and 1. Although this representation for missing measurement is simple, it is useful for many practical systems, especially for network communication systems. For example, if a receiver can receive the measurement data from a transmitter, then γ_k is chosen to be 1, otherwise γ_k is 0.

It is assumed that the process and measurement noises are confined to specified ellipsoidal sets

$$\mathcal{W}_k = \{\boldsymbol{w}_k : w_k^{\mathrm{T}} Q_k^{-1} \boldsymbol{w}_k \leq 1\} \tag{5}$$

$$\mathcal{V}_k = \{\boldsymbol{v}_k : v_k^{\mathrm{T}} R_k^{-1} \boldsymbol{v}_k \leq 1\} \tag{6}$$

where $\boldsymbol{Q}_k = Q_k^{\mathrm{T}} > 0$ and $\boldsymbol{R}_k = R_k^{\mathrm{T}} > 0$ are known matrices with compatible dimensions. The initial state x_0 belongs to a given ellipsoid

$$(x_0 - \hat{x}_0)^{\mathrm{T}} P_0^{-1}(x_0 - \hat{x}_0) \leq 1 \tag{7}$$

where $\hat{x}_0$ is an estimate of x_0 which is assumed to be given, and $\boldsymbol{P}_0 = P_0^{\mathrm{T}} > 0$ is a known matrix.

doi: 10.1049/iet-spr.2009.0244

In this paper, a filter is considered for the uncertain system (1)–(4) which is of the form [29]

$$\hat{x}_{k+1} = \boldsymbol{G}_k\hat{x}_k + \boldsymbol{H}_k\boldsymbol{u}_k + \boldsymbol{L}_k y_k \tag{8}$$

where $\hat{x}_k \in \mathbb{R}^n$ is the state estimate of $\boldsymbol{x}_k$ and G_k, H_k and L_k are the filter parameters to be determined.

Our purpose is to determine an ellipsoid for the state $\boldsymbol{x}_k$, given the missing measurement information y_k at the time instant k for all uncertain matrices $(\boldsymbol{A}_k(\alpha), \boldsymbol{B}_k(\alpha), \boldsymbol{F}_k(\alpha), \boldsymbol{C}_k(\alpha), \boldsymbol{D}_k(\alpha)) \in \Omega_1 + \Omega_2$, the process noise $\boldsymbol{w}_k \in \mathcal{W}_k$ and the measurement noise $\boldsymbol{v}_k \in \mathcal{V}_k$. In other words, we look for $\boldsymbol{P}_k$ and $\hat{x}_k$ such that

$$(\boldsymbol{x}_k - \hat{x}_k)^{\mathrm{T}} P_k^{-1}(\boldsymbol{x}_k - \hat{x}_k) \le 1 \tag{9}$$

subject to $(A_k(\alpha), B_k(\alpha), F_k(\alpha), C_k(\alpha), D_k(\alpha)) \in \Omega_1 + \Omega_2$, $\boldsymbol{w}_k \in \mathcal{W}_k$ and $\boldsymbol{v}_k \in \mathcal{V}_k$.

The above filtering problem is referred to as the robust set-membership filtering problem with missing measurement.

3 Robust set-membership filter design with missing measurement

In this section, a robust set-membership filter is designed for discrete time-varying polytopic uncertain systems (1) and (3) subject to process and measurement noises belonging to the specifying ellipsoidal noise bounds in (5) and (6).

The following theorem provides a method to compute the state estimation ellipsoid for the polytopic uncertain systems where the true state resides and the convex combination approach proposed by [30–32] is applied to compute the state estimation ellipsoid.

Theorem 1: For the polytopic uncertain system (1) and (3) whose parameters reside in polytopes Ω_1 and Ω_2 with given vertices $A_k^{(i)}$, $B_k^{(i)}$, $F_k^{(i)}$, $C_k^{(i)}$ and $D_k^{(i)}$ $(i = 1, 2, \ldots, N)$, N is the number of the vertices. Given that the state $\boldsymbol{x}_k$ belongs to its state estimation ellipsoid $(\boldsymbol{x}_k - \hat{x}_k)^{\mathrm{T}} P_k^{-1}(\boldsymbol{x}_k - \hat{x}_k) \le 1$, where $\hat{x}_k$ and $\boldsymbol{P}_k > 0$ are known, then the one-step ahead state $\boldsymbol{x}_{k+1}$ resides in its state estimation ellipsoid $(\boldsymbol{x}_{k+1} - \hat{x}_{k+1})^{\mathrm{T}} P_{k+1}^{-1}(\boldsymbol{x}_{k+1} - \hat{x}_{k+1}) \le 1$, if there exist $\boldsymbol{P}_{k+1} > 0$, G_k, H_k, L_k, $\tau_1 > 0$, $\tau_2 > 0$, $\tau_3 > 0$ such that the following linear matrix inequality (see (10))

holds, where (see (11))

and $A_k^{(i)}$, $B_k^{(i)}$, $F_k^{(i)}$, $C_k^{(i)}$ and $D_k^{(i)}$ are the matrices in (2) and (4) at the ith vertex of the polytope. Moreover, $\hat{x}_{k+1}$ is determined by

$$\hat{x}_{k+1} = G_k\hat{x}_k + H_k\boldsymbol{u}_k + L_k y_k \tag{12}$$

Proof: The one-step ahead estimation error $\boldsymbol{x}_{k+1} - \hat{x}_{k+1}$ from (1) and (8) is written as

$$\begin{aligned}
\boldsymbol{x}_{k+1} - \hat{x}_{k+1} &= \boldsymbol{A}_k(\alpha)\boldsymbol{x}_k + \boldsymbol{F}_k(\alpha)\boldsymbol{u}_k + \boldsymbol{B}_k(\alpha)\boldsymbol{w}_k \\
&\quad - G_k\hat{x}_k - H_k\boldsymbol{u}_k - L_k y_k \\
&= \boldsymbol{A}_k(\alpha)\boldsymbol{x}_k + \boldsymbol{F}_k(\alpha)\boldsymbol{u}_k + \boldsymbol{B}_k(\alpha)\boldsymbol{w}_k - G_k\hat{x}_k \\
&\quad - H_k\boldsymbol{u}_k - L_k[\gamma_k(\boldsymbol{C}_k(\alpha)\boldsymbol{x}_k + D_k(\alpha)\boldsymbol{v}_k)] \\
&= (\boldsymbol{A}_k(\alpha) - \gamma_k L_k\boldsymbol{C}_k(\alpha))\boldsymbol{x}_k + \boldsymbol{F}_k(\alpha)\boldsymbol{u}_k \\
&\quad + \boldsymbol{B}_k(\alpha)\boldsymbol{w}_k - G_k\hat{x}_k - H_k\boldsymbol{u}_k - \gamma_k L_k D_k(\alpha)\boldsymbol{v}_k
\end{aligned} \tag{13}$$

Since $(\boldsymbol{x}_k - \hat{x}_k)^{\mathrm{T}} P_k^{-1}(\boldsymbol{x}_k - \hat{x}_k) \le 1$, there exists a z with $\|z\| \le 1$ such that

$$\boldsymbol{x}_k = \hat{x}_k + E_k z \tag{14}$$

where E_k is a factorisation of $\boldsymbol{P}_k = E_k E_k^{\mathrm{T}}$.

Substituting (14) into (13) yields

$$\begin{aligned}
&\boldsymbol{x}_{k+1} - \hat{x}_{k+1} \\
&= (\boldsymbol{A}_k(\alpha) - \gamma_k L_k\boldsymbol{C}_k(\alpha))\hat{x}_k - G_k\hat{x}_k + \boldsymbol{F}_k(\alpha)\boldsymbol{u}_k - H_k\boldsymbol{u}_k \\
&\quad + (\boldsymbol{A}_k(\alpha) - \gamma_k L_k\boldsymbol{C}_k(\alpha))E_k z + \boldsymbol{B}_k(\alpha)\boldsymbol{w}_k - \gamma_k L_k D_k(\alpha)\boldsymbol{v}_k
\end{aligned} \tag{15}$$

Denoting

$$\eta = \begin{bmatrix} 1 \\ z \\ \boldsymbol{w}_k \\ \boldsymbol{v}_k \end{bmatrix} \tag{16}$$

and (see (17))

we can rewrite (15) as follows

$$\boldsymbol{x}_{k+1} - \hat{x}_{k+1} = \Pi(\hat{x}_k)\eta \tag{18}$$

Thus, $(\boldsymbol{x}_{k+1} - \hat{x}_{k+1})^{\mathrm{T}} P_{k+1}^{-1}(\boldsymbol{x}_{k+1} - \hat{x}_{k+1}) \le 1$ can be written as

$$\eta^{\mathrm{T}}\Pi(\hat{x}_k)^{\mathrm{T}} P_{k+1}^{-1}\Pi(\hat{x}_k)\eta - \eta^{\mathrm{T}}\mathrm{diag}(1, 0, 0, 0)\eta \le 0 \tag{19}$$

Now $\|z\| \le 1$, $w_k^{\mathrm{T}} Q_k^{-1}\boldsymbol{w}_k \le 1$, and $v_k^{\mathrm{T}} R_k^{-1}\boldsymbol{v}_k \le 1$ are also written as

$$\eta^{\mathrm{T}}\mathrm{diag}(-1, I, 0, 0)\eta \le 0 \tag{20}$$

$$\eta^{\mathrm{T}}\mathrm{diag}(-1, 0, Q_k^{-1}, 0)\eta \le 0 \tag{21}$$

$$\eta^{\mathrm{T}}\mathrm{diag}(-1, 0, 0, R_k^{-1})\eta \le 0 \tag{22}$$

According to S-Procedure [27, 33], the sufficient condition

$$\begin{bmatrix} -\boldsymbol{P}_{k+1} & \Pi^{(i)}(\hat{x}_k, \boldsymbol{u}_k) \\ (\Pi^{(i)}(\hat{x}_k, \boldsymbol{u}_k))^{\mathrm{T}} & \mathrm{diag}(-1 + \tau_1 + \tau_2 + \tau_3,\ -\tau_1 I,\ -\tau_2 Q_k^{-1},\ -\tau_3 R_k^{-1}) \end{bmatrix} \le 0, \quad i = 1, 2, \ldots, N \tag{10}$$

$$\Pi^{(i)}(\hat{x}_k, \boldsymbol{u}_k) = [(A_k^{(i)} - \gamma_k L_k C_k^{(i)})\hat{x}_k - G_k\hat{x}_k + F_k^{(i)}\boldsymbol{u}_k - H_k\boldsymbol{u}_k \quad (A_k^{(i)} - \gamma_k L_k C_k^{(i)})E_k \quad B_k^{(i)} \quad -\gamma_k L_k D_k^{(i)}]\, i = 1, 2, \ldots, N \tag{11}$$

$$\Pi(\hat{x}_k, \boldsymbol{u}_k) = [(\boldsymbol{A}_k(\alpha) - \gamma_k L_k\boldsymbol{C}_k(\alpha))\hat{x}_k - G_k\hat{x}_k + \boldsymbol{F}_k(\alpha)\boldsymbol{u}_k - H_k\boldsymbol{u}_k \quad (\boldsymbol{A}_k(\alpha) - \gamma_k L_k\boldsymbol{C}_k(\alpha))E_k \quad B_k(\alpha) \quad -\gamma_k L_k D_k(\alpha)] \tag{17}$$

such that the inequalities (20)–(22) imply (19) to hold is that there exist positive scalars τ_1, τ_2 and τ_3 such that

$$\Pi(\hat{x}_k)^{\mathrm{T}} P_{k+1}^{-1} \Pi(\hat{x}_k) - \mathrm{diag}(1, 0, 0, 0) - \tau_1 \mathrm{diag}(-1, I, 0, 0) - \tau_2 \mathrm{diag}(-1, 0, Q_k^{-1}, 0) - \tau_3 \mathrm{diag}(-1, 0, 0, R_k^{-1}) \leq 0 \tag{23}$$

Equation (23) is written in the following compact form

$$\mathrm{diag}(-1 + \tau_1 + \tau_2 + \tau_3,\ -\tau_1 I,\ -\tau_2 Q_k^{-1},\ -\tau_3 R_k^{-1}) + \Pi(\hat{x}_k)^{\mathrm{T}} P_{k+1}^{-1} \Pi(\hat{x}_k) \leq 0 \tag{24}$$

By using Schur complements in [27], (24) is equivalent to (see (25))

Now, we prove the equivalence between (25) and (10) for all $i \in \{1, 2, \ldots, N\}$. The proof from (25) to (10) is straightforward since if (25) is satisfied for all the polytope it must be the case at all the vertices. The proof from (10) to (25) is deduced from (2), (4) and (10) (see (26)).

by noting that $\alpha_i \geq 0$ and $\sum_{i=1}^{N} \alpha_i = 1$. □

Theorem 1 outlines the principle of determining the current state estimation ellipsoid given the previous state estimation ellipsoid. However, it does not provide an optimal state estimation ellipsoid. Next, we apply the convex optimisation approach to determine an optimal ellipsoid. $\boldsymbol{P}_{k+1}$ is obtained by solving the following optimisation problem

$$\min_{\boldsymbol{P}_{k+1}>0, G_k, H_k, L_k, \tau_1>0, \tau_2>0, \tau_3>0} \mathrm{trace}(\boldsymbol{P}_{k+1}) \quad \text{subject to (10)} \tag{27}$$

Equation (27) provides the computation of the state estimation ellipsoid of minimal size in the sense of trace.

Remark 3: We can see from Theorem 1 that the inequality (10) is linear with respect to the variables $\boldsymbol{P}_{k+1}$, G_k and L_k, τ_1, τ_2 and τ_3. Hence, the optimisation problem (27) subject to (10) can be solved by the existing semi-definite programming (SDP) via an interior-point approach [34, 35].

Remark 4: The trace of $\boldsymbol{P}_{k+1}$ is optimised at each time step in an effort to find the smallest ellipsoid for the state estimate. Other measures of the ellipsoid can also be introduced, for example, determinant [14, 36].

Now we summarise a recursive algorithm for the set-membership filtering as follows:

The robust set-membership filtering recursive algorithm

Step 1: Start with the initial values ($\hat{x}_0$ and $\boldsymbol{P}_0$). Set the recursive times K and $k = 0$;
Step 2: Find the shape of the state estimation ellipsoid $\boldsymbol{P}_{k+1}$ and filter parameters G_k, H_k and L_k by solving the optimisation problem (27);
Step 3: Compute the state estimate $\hat{x}_{k+1}$ by using (12);
Step 4: If $k = K$, then Stop, otherwise, $k = k + 1$ and go to Step 2.

4 Simulation results

Consider a radar tracking system

$$\boldsymbol{x}_{k+1} = \begin{bmatrix} 0.9 + \alpha & T \\ 0 & 0.9 \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \boldsymbol{u}_k + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} \boldsymbol{w}_k \tag{28}$$

where T is the sampling time. The state is $\boldsymbol{x}_k = [s(k) \quad \dot{s}(k)]^{\mathrm{T}}$, where $s(k)$ and $\dot{s}(k)$ are the position and velocity of the target at time kT, respectively. Owing to modelling errors, α is the unknown but it belongs to the known interval $[\alpha_{\min}, \alpha_{\max}]$.

The measurement model with missing measurement is described as

$$y_k = \gamma_k \left(\begin{bmatrix} 0.5 & 1 \end{bmatrix} \boldsymbol{x}_k + \boldsymbol{v}_k \right) \tag{29}$$

We consider this system over a finite-time interval of $K = 100$ samples. Furthermore, we assume that the measured signal $y(k)$ is unavailable at the following time instants

$$k = 5, 6, 7, 51, 52, 53, 81, 82, 83$$

that is, at the above time instants, $\gamma_k = 0$, otherwise $\gamma_k = 1$.

In the simulation, the sample period T is chosen as 0.03 s, $\boldsymbol{u}_k = 0.1$, $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are non-Gaussian noise with Cauchy distribution. In both pdfs the initial state is set as $x_0 = [8 - 10]^{\mathrm{T}}$, which belongs to the ellipsoid $(x_0 - \hat{x}_0)^{\mathrm{T}} P_0^{-1} (x_0 - \hat{x}_0) \leq 1$, where $\hat{x}_0 = [7 - 9]^{\mathrm{T}}$ and $\boldsymbol{P}_0 = \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}$, $\boldsymbol{Q}_k = 1$ and $\boldsymbol{R}_k = 1$.

We first consider the known system, that is, $\alpha = 0$ in (28). The simulation results are obtained by solving the convex optimisation problem in (27) subject to (10) under Matlab 6.5 with YALMIP 3.0 and SeDuMi 1.1 [37]. YALMIP is a modelling language for advanced modelling and solution of convex and non-convex optimisation problems, and SeDuMi is an interface for the self-dual-minimisation package developed by Jos F. Sturm [38]. Both are user-friendly free Matlab packages. Fig. 1 shows the phase-plane estimation using the proposed set-membership filter. It can

$$\begin{bmatrix} -\boldsymbol{P}_{k+1} & \Pi(\hat{x}_k, \boldsymbol{u}_k) \\ \Pi(\hat{x}_k, \boldsymbol{u}_k)^{\mathrm{T}} & \mathrm{diag}(-1 + \tau_1 + \tau_2 + \tau_3,\ -\tau_1 I,\ -\tau_2 Q_k^{-1},\ -\tau_3 R_k^{-1}) \end{bmatrix} \leq 0 \tag{25}$$

$$\begin{bmatrix} -\boldsymbol{P}_{k+1} & \Pi(\hat{x}_k, \boldsymbol{u}_k) \\ (\Pi(\hat{x}_k, \boldsymbol{u}_k))^{\mathrm{T}} & \mathrm{diag}(-1 + \tau_1 + \tau_2 + \tau_3,\ -\tau_1 I,\ -\tau_2 Q_k^{-1},\ -\tau_3 R_k^{-1}) \end{bmatrix} = \sum_{i=1}^{N} \alpha_i \begin{bmatrix} -\boldsymbol{P}_{k+1} & \Pi^{(i)}(\hat{x}_k, \boldsymbol{u}_k) \\ (\Pi^{(i)}(\hat{x}_k, \boldsymbol{u}_k))^{\mathrm{T}} & \mathrm{diag}(-1 + \tau_1 + \tau_2 + \tau_3,\ -\tau_1 I,\ -\tau_2 Q_k^{-1},\ -\tau_3 R_k^{-1}) \end{bmatrix} \leq 0 \tag{26}$$

IET Signal Process., 2012, Vol. 6, Iss. 4, pp. 341–347
doi: 10.1049/iet-spr.2009.0244

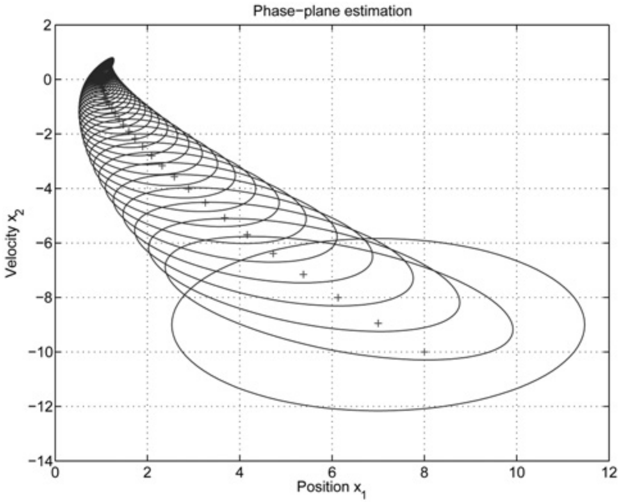

Fig. 1 *Phase-plane estimation using the proposed set-membership filter for the known system*

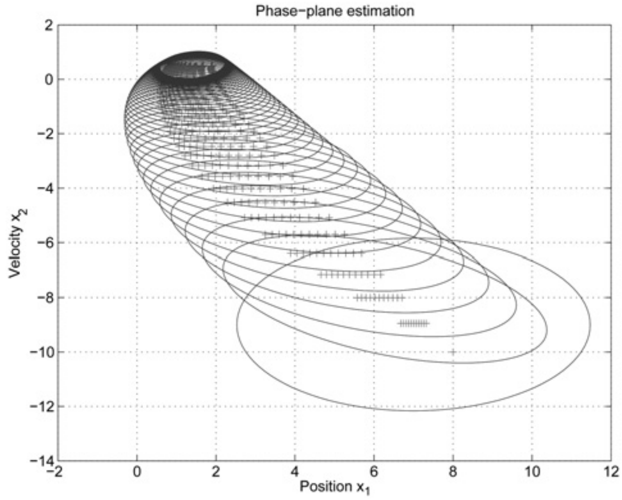

Fig. 4 *Phase-plane estimation using the proposed set-membership filter for the polytopic uncertain system*

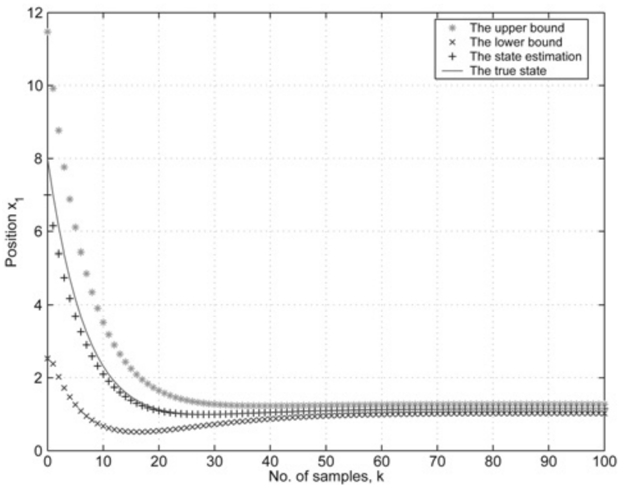

Fig. 2 *True state value, state estimation and its bounds with the known system for target position*

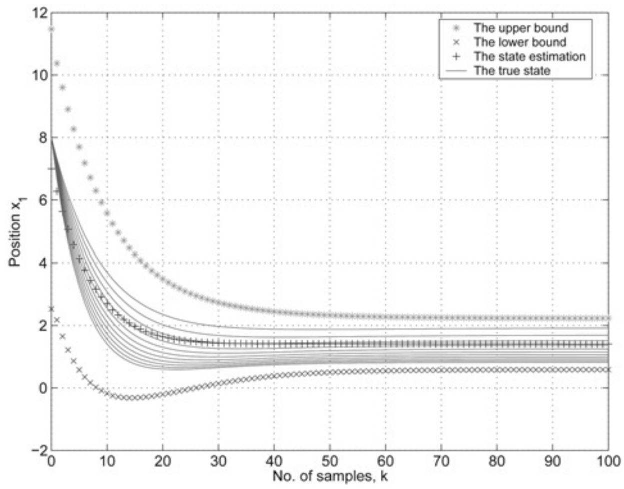

Fig. 5 *True state value, state estimation and its bounds with the polytopic uncertain system for target position*

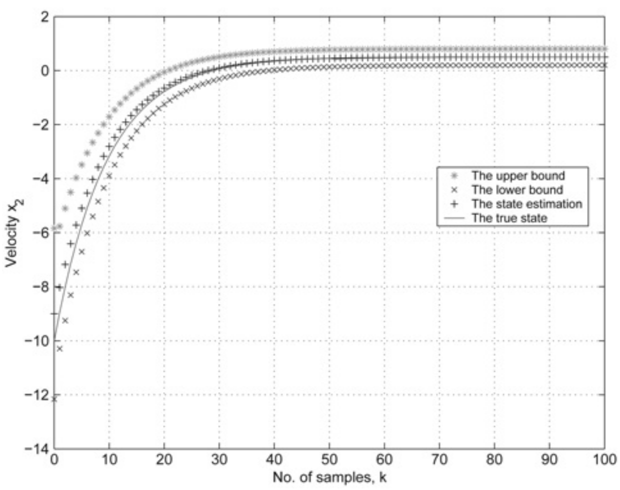

Fig. 3 *True state value, state estimation and its bounds with the known system for target velocity*

be seen that the estimated ellipsoids always contain the true states. Figs. 2 and 3 further confirm that the true position and true velocity reside between the upper bound and lower bound.

Next, we consider the polytopic uncertain system, that is, $\alpha \in [-0.04,\ 0.04]$ in (28). The simulation results are obtained by solving the convex optimisation problem in (27) subject to (10) under Matlab 6.5 with YALMIP 3.0 and SeDuMi 1.1 [37]. Fig. 4 shows phase-plane estimation using the proposed set-membership filter. Owing to the uncertain parameters α, the true states distribute in the phase-plane with respect to different α. However, they are always contained in the estimated ellipsoid. It is also shown from Figs. 5 and 6 that the true position and true velocity reside between the upper bound and lower bound for all uncertain parameters α. At any time step, we always know that the target belongs to the estimated region. The target can be fully tracked. Therefore our proposed filtering algorithm is useful for target tracking and attacking the target, even though the target systems are not exactly modelled.

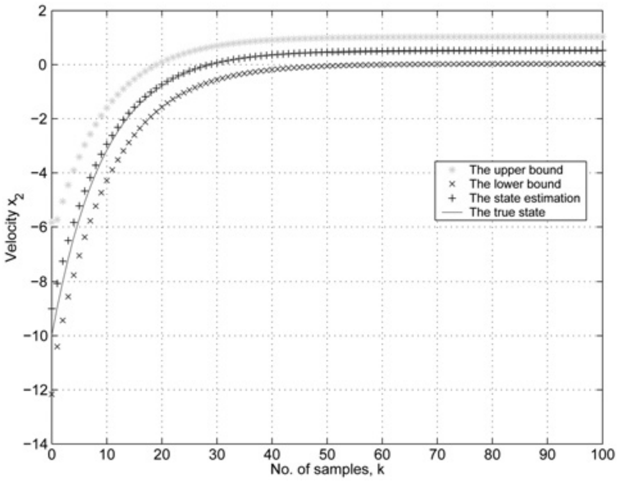

Fig. 6 *True state value, state estimation and its bounds with the polytopic uncertain system for target velocity*

From the simulations, we can see that the bounds without parameter uncertainty are narrower than the bounds with parameter uncertainty. Thus, the uncertainty makes us identify the target in the greater range. Moreover, Figs. 1–6 show that the estimated ellipsoid will become smaller. Hence, the proposed algorithm shows the ability of convergence.

5 Conclusions

This paper has provided a new approach that is able to deal with missing measurement, polytopic uncertainties and non-Gaussian unknown-but-bounded process and measurement noises in filtering problem for discrete time-varying systems. A robust set-membership filter has been developed and a recursive algorithm has been derived to estimate a state ellipsoid which always contains the state. An optimal possible estimate set is computed recursively by solving the semi-definite programming problems. An illustrative example has demonstrated the feasibility of the proposed filtering methods. The algorithm is computationally attractive for on-line systems with missing measurement and polytopic uncertainties in the presence of unknown-but-bounded process and measurement noises. In the polytopic system, how to measure the uncertain parameter α and reduce the estimated region is our future research topic. Our method can also be extended to other kinds of systems, for example, continuous-time systems and time-delay systems. They will be one of our future research topics. The much more challenging research topics should be the study of the convergence of the algorithms and how to reduce the conservatism of the possible estimation sets. Another challenging work is to consider the missing measurement with stochastic binary switching sequence.

6 Acknowledgment

This work has been supported by the Engineering and Physical Sciences Research Council (EPSRC) of the U.K. under Grant No. EP/C007654/1, the Research Development and Incentives Program of Central Queensland University under Merit Grant RSH/2028, the National Nature Science Foundation of China under Grant No. 61174064, the National Program on Key Basic Research Project under Grant 2012CB720502, the Science and Technology Commission of Shanghai Municipality under Grant No. 10PJ1402600 and the Fundamental Research Funds for the Central Universities under Grant WJ0913001. The authors would like to thank the anonymous reviewers for their helpful comments and suggestions which helped improve the paper in many aspects.

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doi: 10.1049/iet-spr.2009.0244

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