

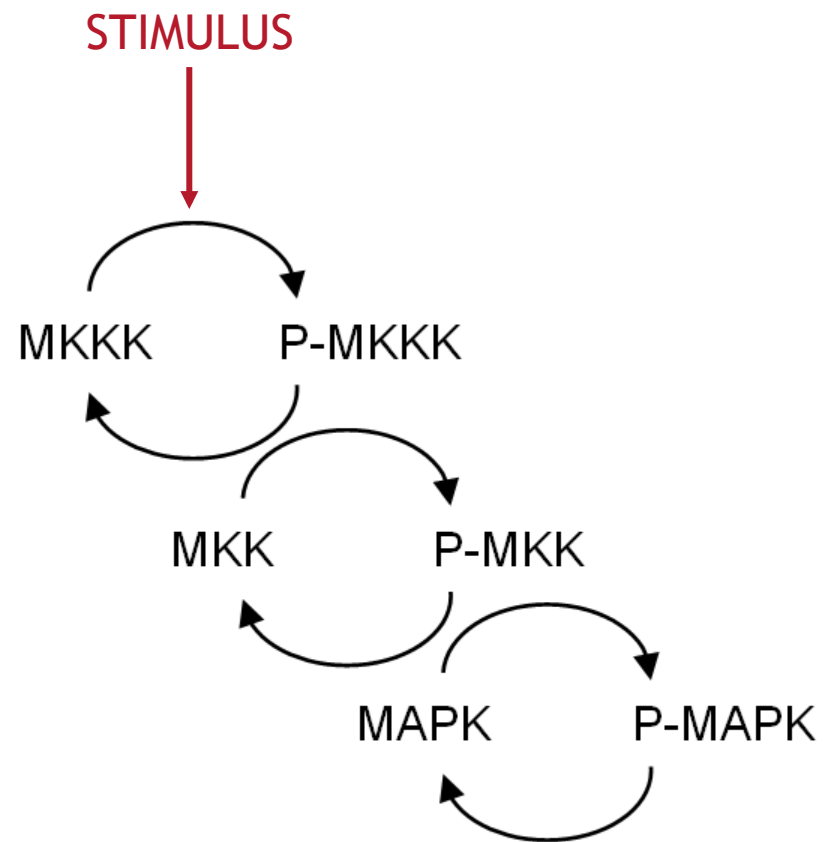
A structured approach ...
Part III
Biological applications

David Gilbert

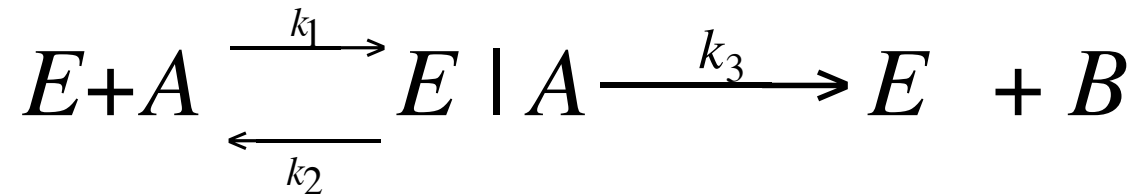
Bioinformatics Research Centre University of Glasgow, Glasgow, UK

MAPK Pathway

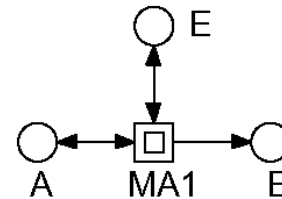
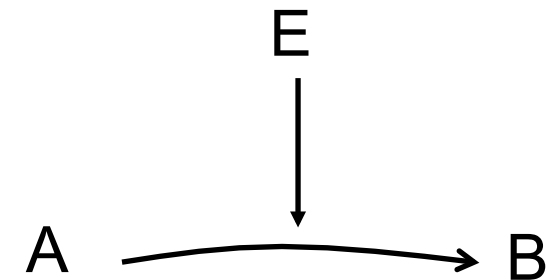
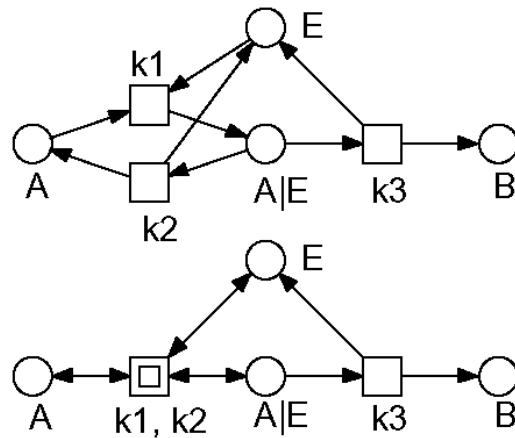
- Responds to wide range of stimuli: cytokines, growth factors, neurotransmitters, cellular stress and cell adherence,...
- Pivotal role in many key cellular processes:
 - growth control in all its variations,
 - cell differentiation and survival
 - cellular adaptation to chemical and physical stress.
- Deregulated in various diseases: cancer; immunological, inflammatory and degenerative syndromes,
- Represents an important drug target.



MA1: Mass action for enzymatic reaction - phosphorylation

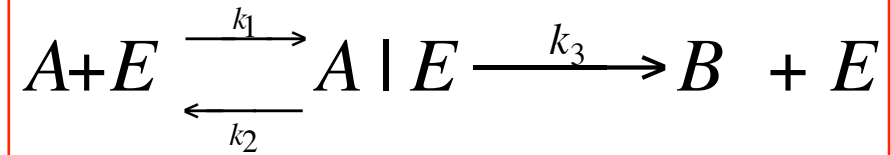


- A: substrate
- B: product (phosphorylated A)
- E: enzyme (kinase)
- E|A substrate-enzyme complex



Differential equations

Enzymatic reaction



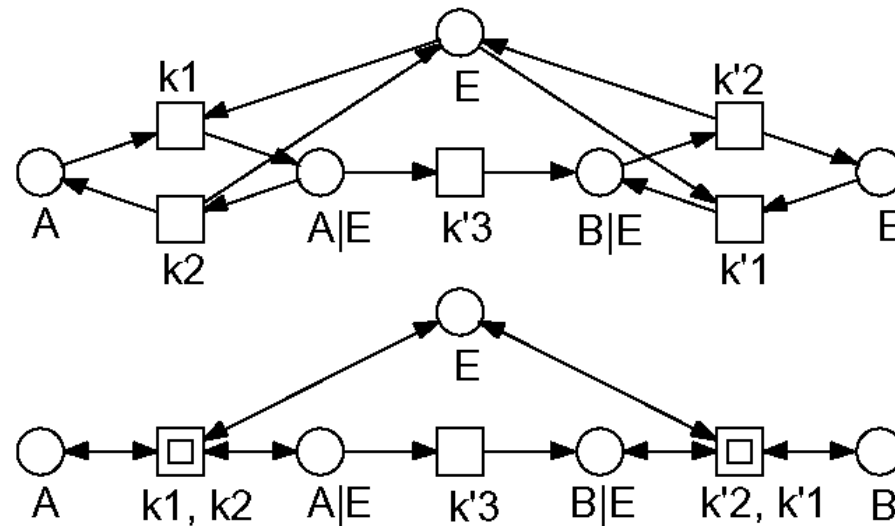
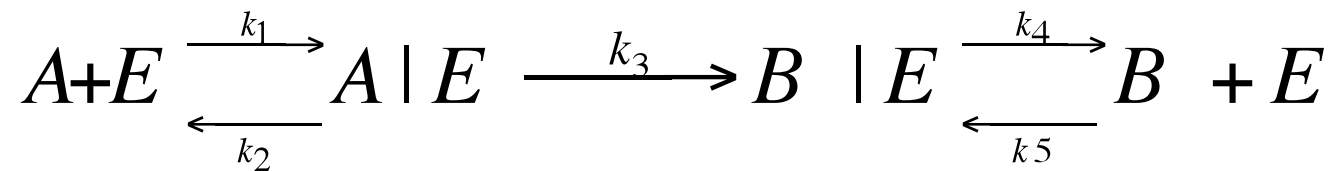
$$\frac{d[A]}{dt} = -k_1 \times [A] \times [E] + k_2 \times [A \mid E]$$

$$\frac{d[A \mid E]}{dt} = +k_1 \times [A] \times [E] - k_2 \times [A \mid E] - k_3 \times [A \mid E]$$

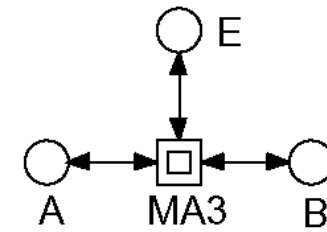
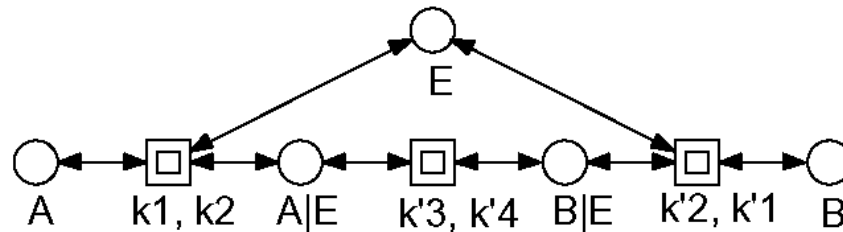
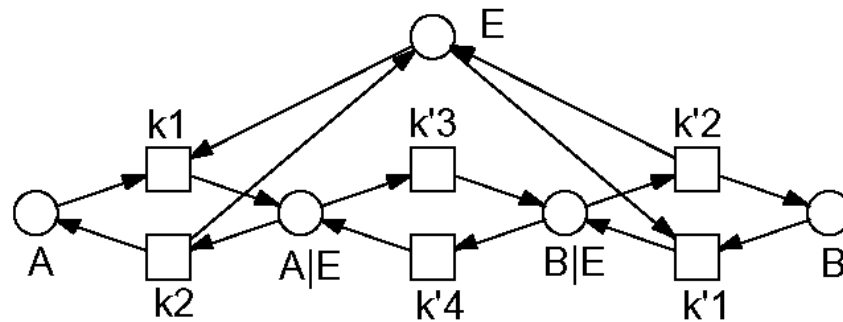
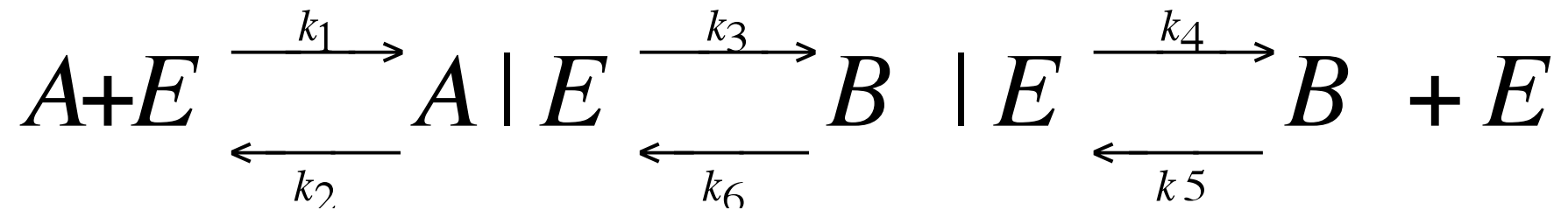
$$\frac{d[B]}{dt} = +k_3 \times [A \mid E]$$

$$\frac{d[E]}{dt} = -k_1 \times [A] \times [E] + k_2 \times [A \mid E] + k_3 \times [A \mid E]$$

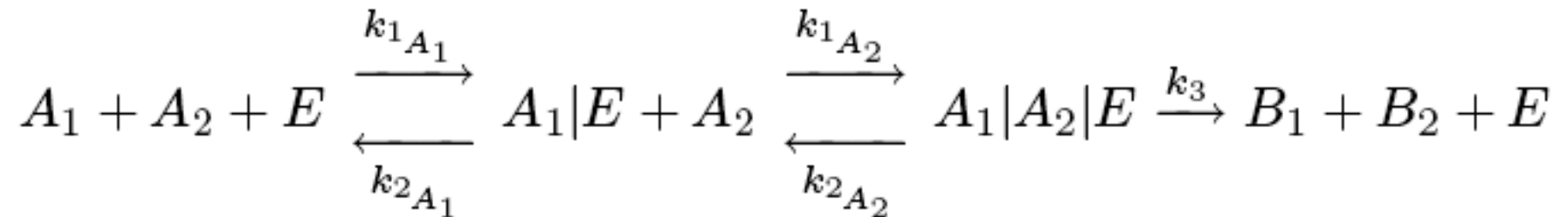
MA2 model



MA3 model

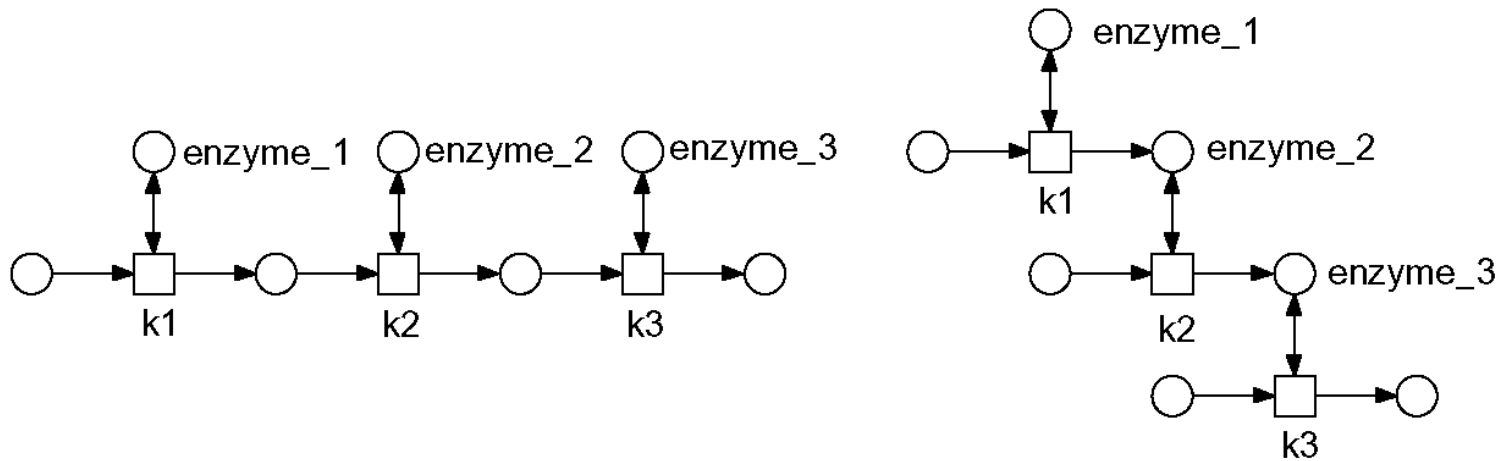


Multiple substrates

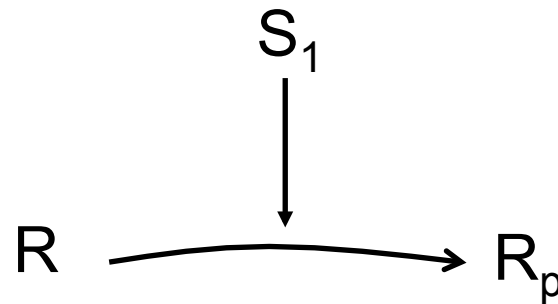


Metabolic pathways vs Signalling Pathways

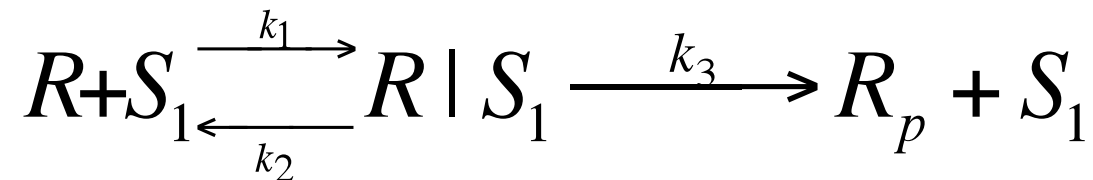
(Petri Nets)



Mass action for enzymatic reaction - phosphorylation



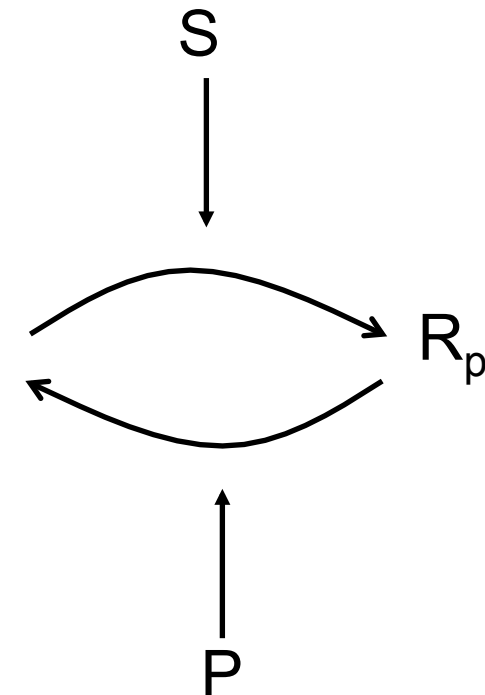
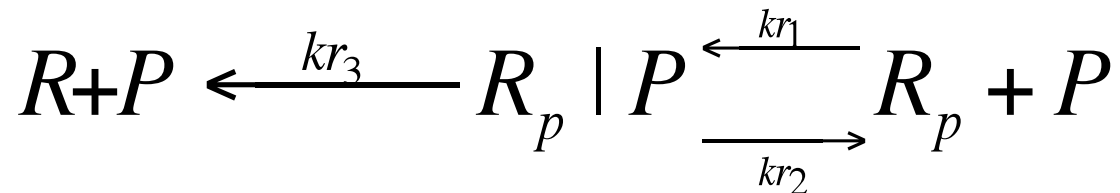
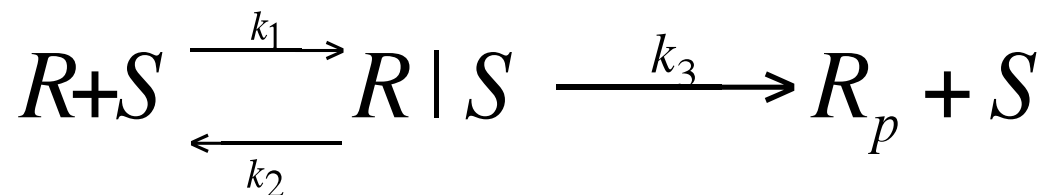
- R: substrate,
- R_p: product (phosphorylated R)
- S₁: enzyme (kinase)
- R|S₁ substrate-enzyme complex



Phosphorylation - dephosphorylation step

Mass action model 1

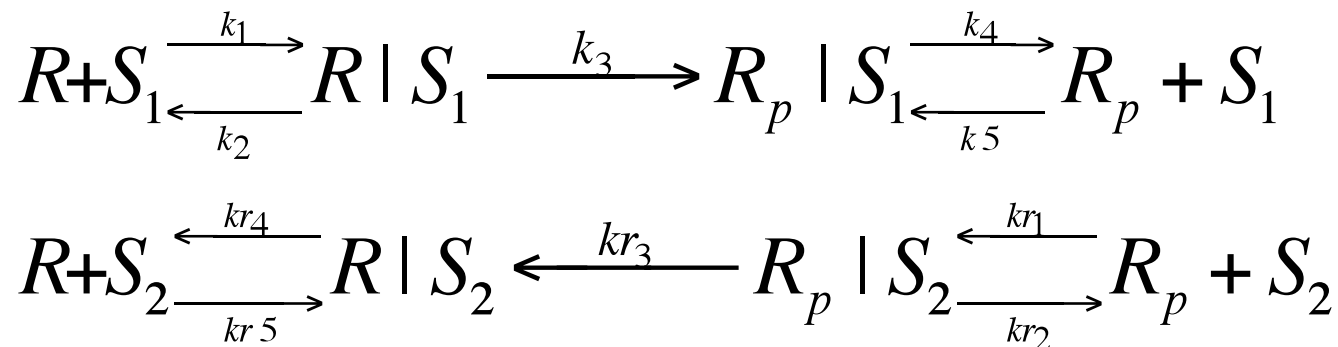
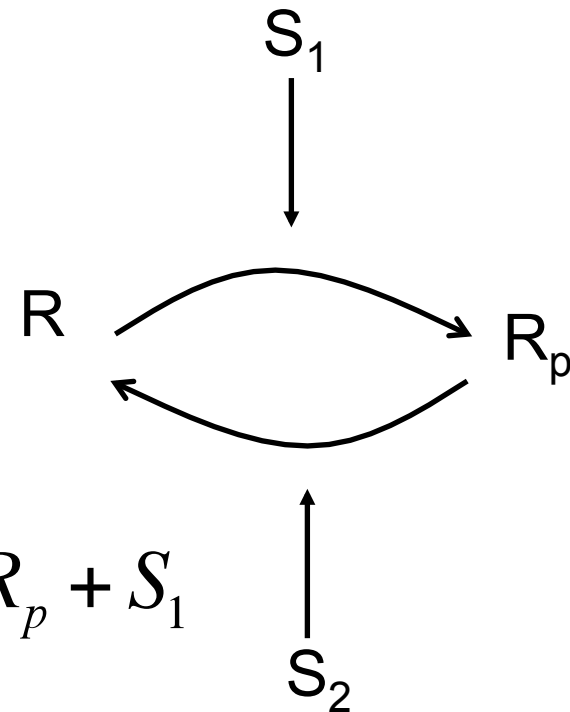
- R: unphosphorylated form
- R_p : phosphorylated form
- S: kinase
- P: phosphatase
- R|S unphosphorylated+kinase complex
- $R|P$ unphosphorylated+phosphatase complex



Phosphorylation - dephosphorylation loop

Mass action model 2

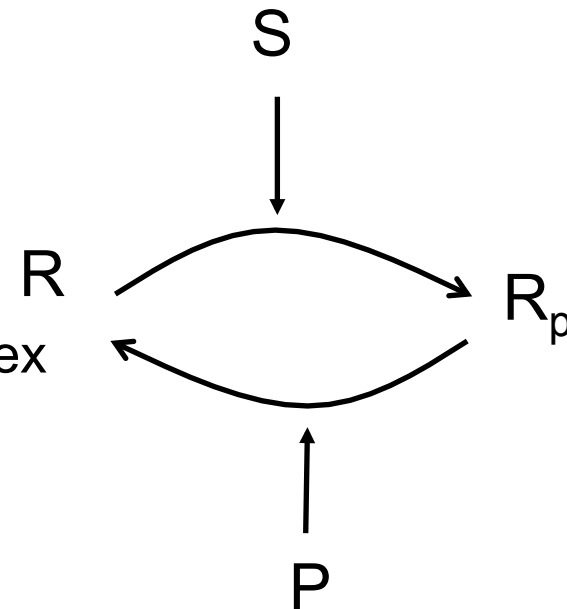
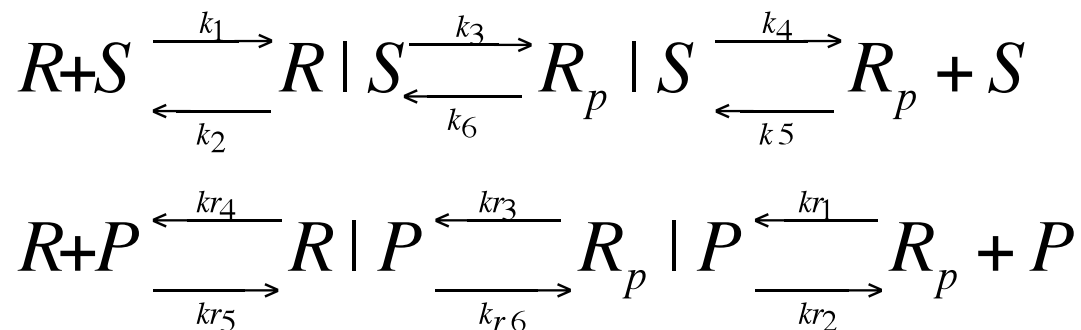
- R: unphosphorylated form
- R_p : phosphorylated form
- S_1 : kinase
- S_2 : phosphatase
- $R|S_1$ unphosphorylated+kinase complex
- $R_p|S_1$ phosphorylated+kinase complex
- $R|S_2$ unphosphorylated+phosphatase complex
- $R_p|S_2$ phosphorylated+phosphatase complex



Phosphorylation - dephosphorylation step

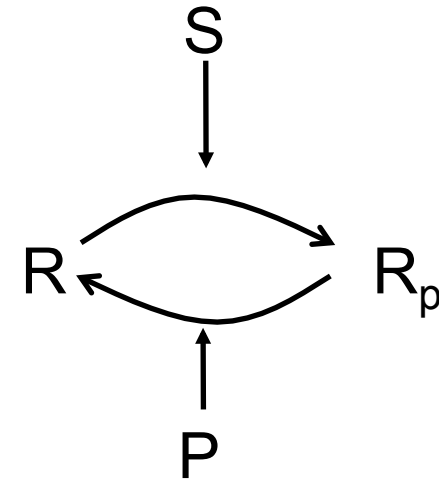
Mass action (all singing/dancing)

- R: unphosphorylated form
- R_p : phosphorylated form
- S: kinase
- P: phosphotase
- R|S unphosphorylated+kinase complex
- R|P unphosphorylated+phosphotase complex



Michaelis-Menten equation for phosphorylation-dephosphorylation

$$V = k_3 \times [S] \times \frac{[R]}{(K_{m1} + [R])} - k_3' \times \frac{[R_p]}{(K_{m2} + [R_p])}$$

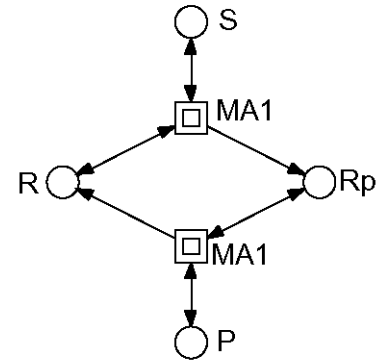
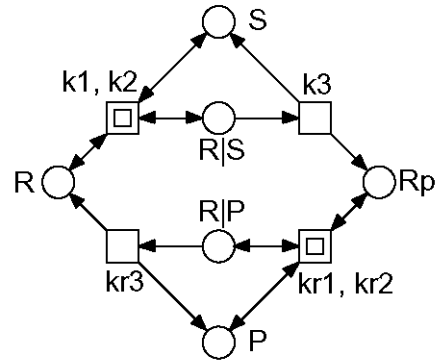


- Assumptions:
 1. No product reverts to initial substrate
 2. MM Equation holds at initial stage of reaction before concentration of product is appreciable
 3. [Enzyme] << [Substrate]
- K_m is [Substrate] at which the reaction rate is half its maximum value
- $dR_p/dt ==$ reaction rate V
- $k_3 \times S == V_{max}$ for the forward reaction
- $k_3' == V_{max}$ for the reverse reaction (Phosphatase is ignored)
- $K_{m1} == (k_2 + k_3)/k_1$ (k's from mass-action 1)

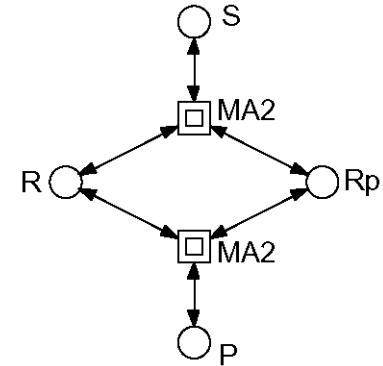
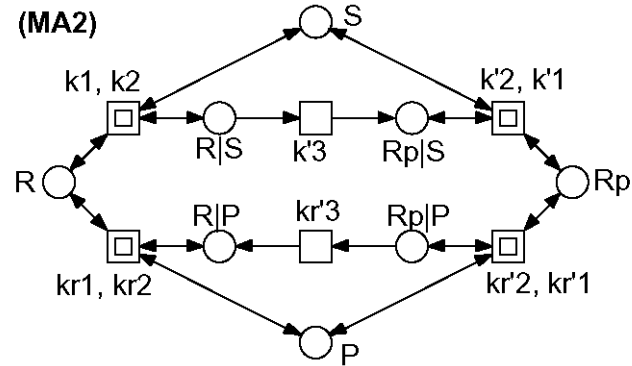
Questions

- Is Michaelis-Menten adequate for phosphorylation pathways?
- Is Mass Action sufficient/correct for these pathways?
- What is the effect of negative feedback?
- Can we confirm the 'negative feedback amplifier' behaviour in both MM and MA models
- Can oscillators be built?
- Overall, what are the rules for component-based construction?

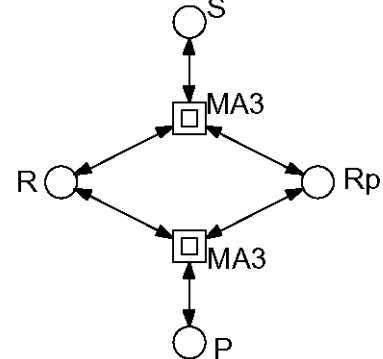
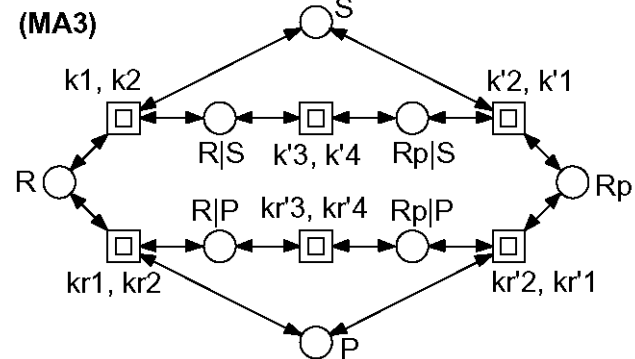
(MA1)



(MA2)

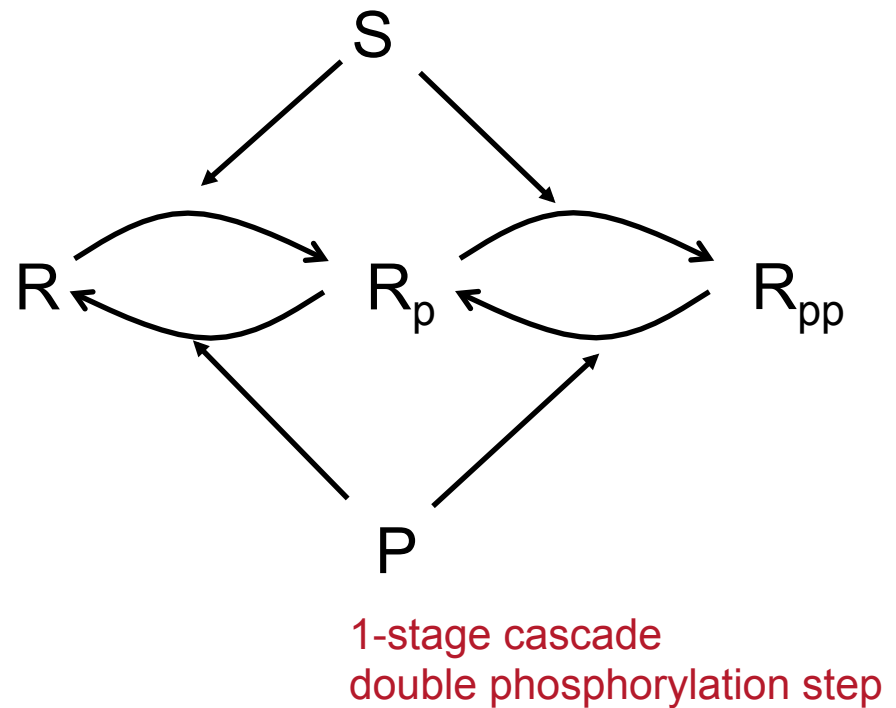
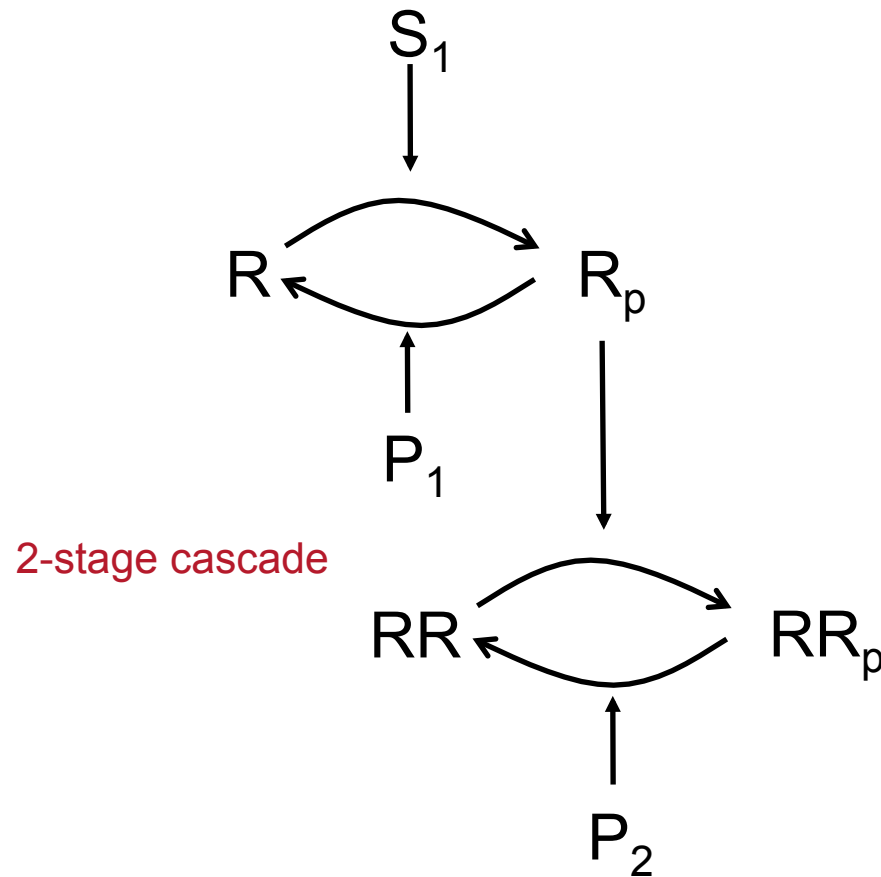


(MA3)



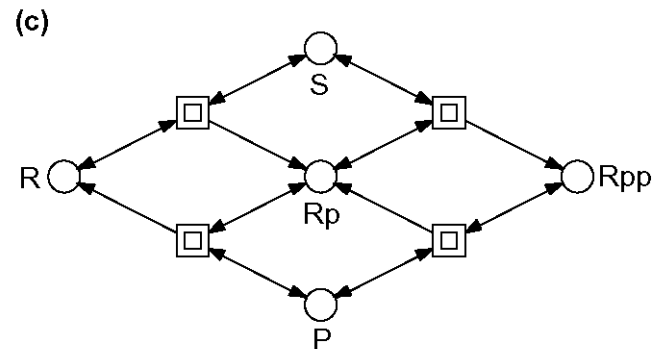
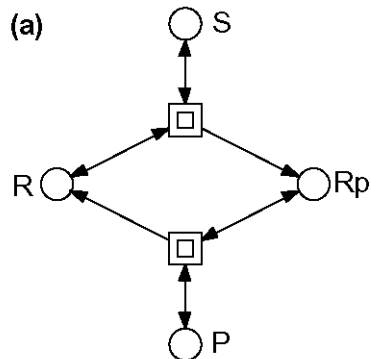
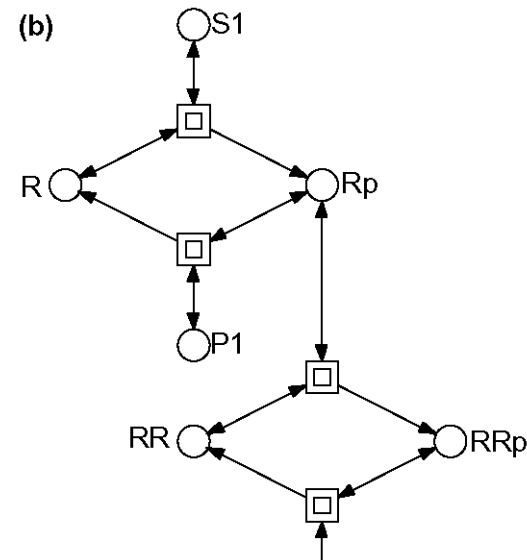
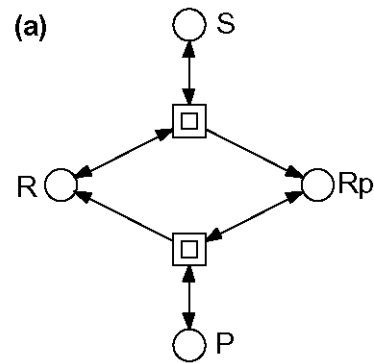
Composition

Vertical & horizontal

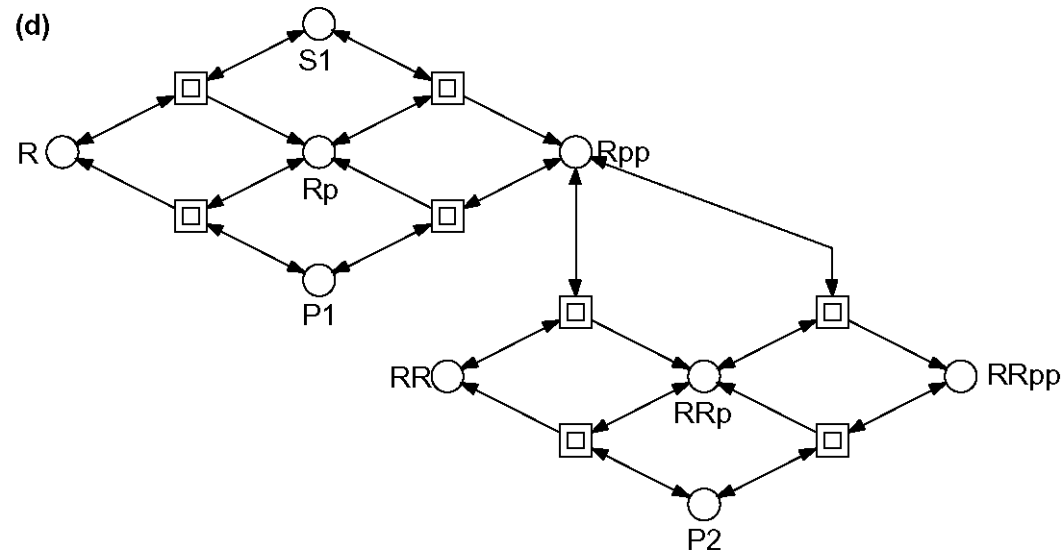


Composition

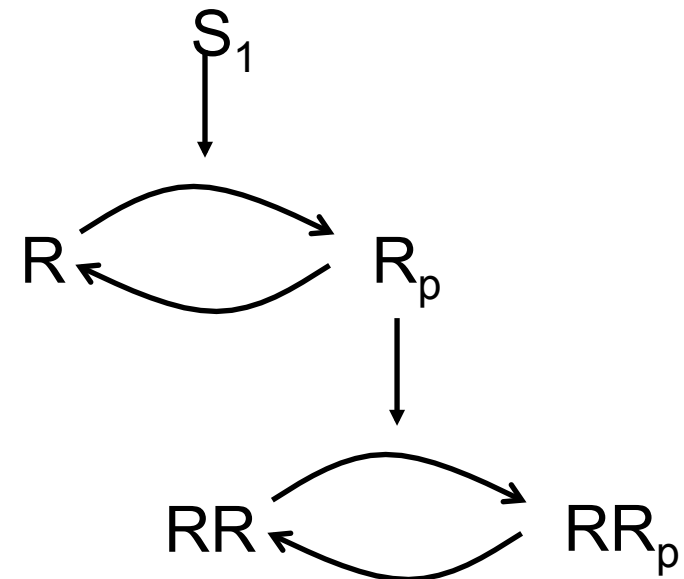
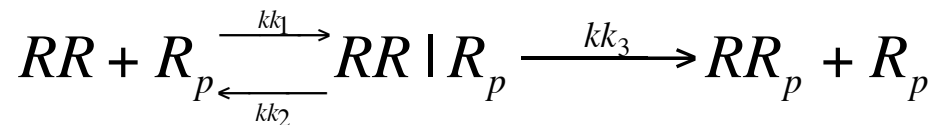
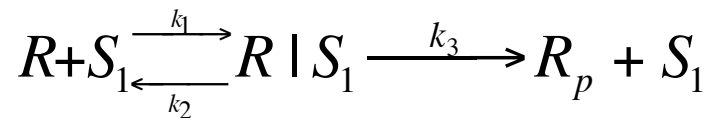
Vertical & horizontal



Two stage, double phosphorylation



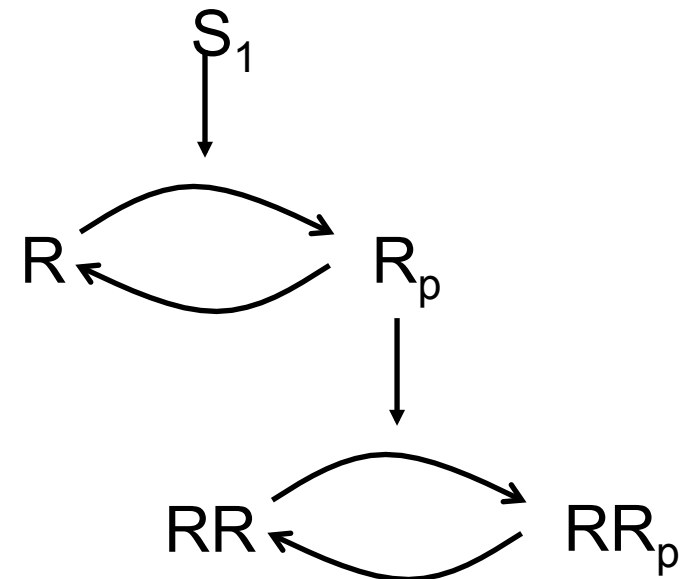
Phosphorylation cascade: 2-stage, Mass Action model 1



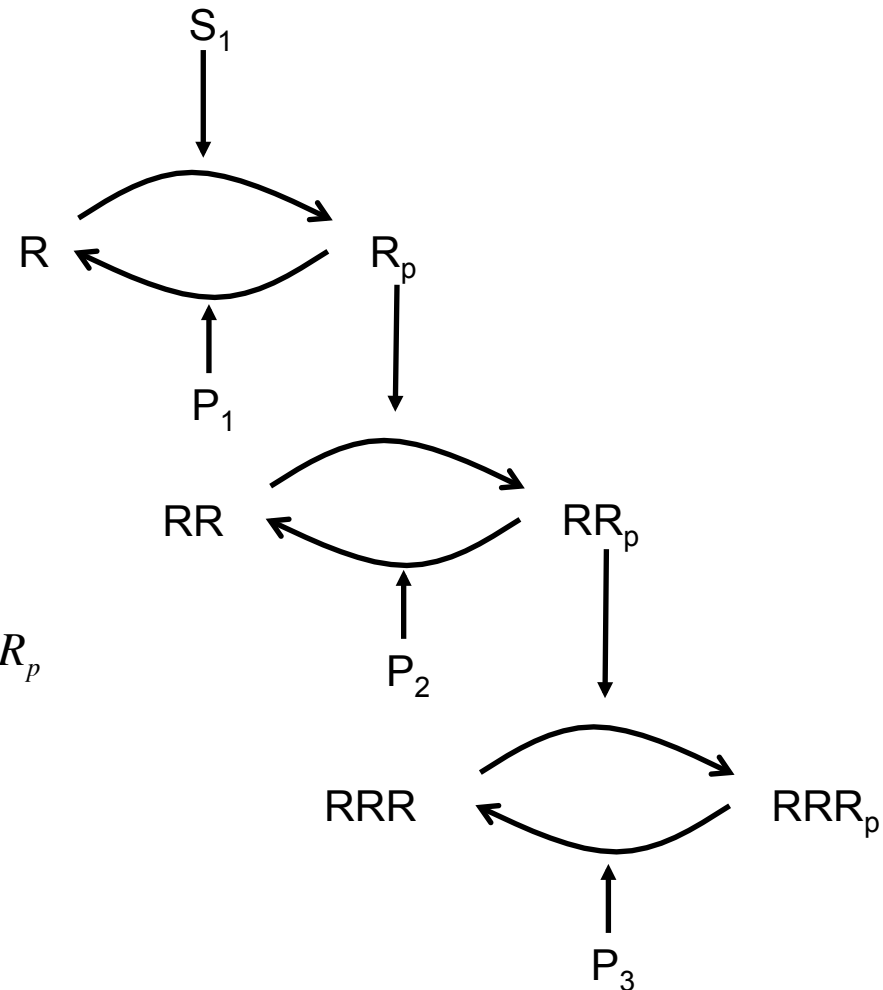
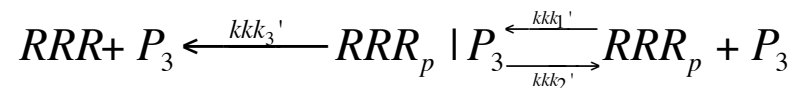
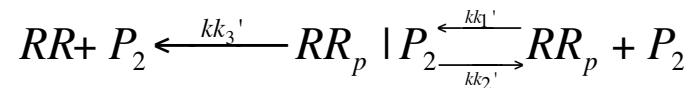
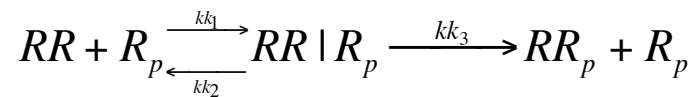
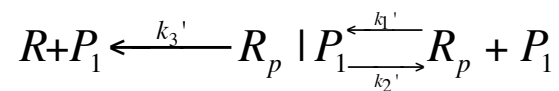
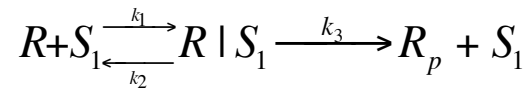
Phosphorylation cascade: 2-stage, Michaelis-Menten

$$\frac{dR_p}{dt} = \frac{k_3 \times S_1 \times R}{K_{m1} + R} - \frac{k_3' \times R_p}{K_{m2} + R_p}$$

$$\frac{dRR_p}{dt} = \frac{kk_3 \times R_p \times RR}{KK_{m1} + RR} - \frac{kk_3' \times RR_p}{KK_{m2} + RR_p}$$



3-stage Phosphorylation cascade (Mass Action)

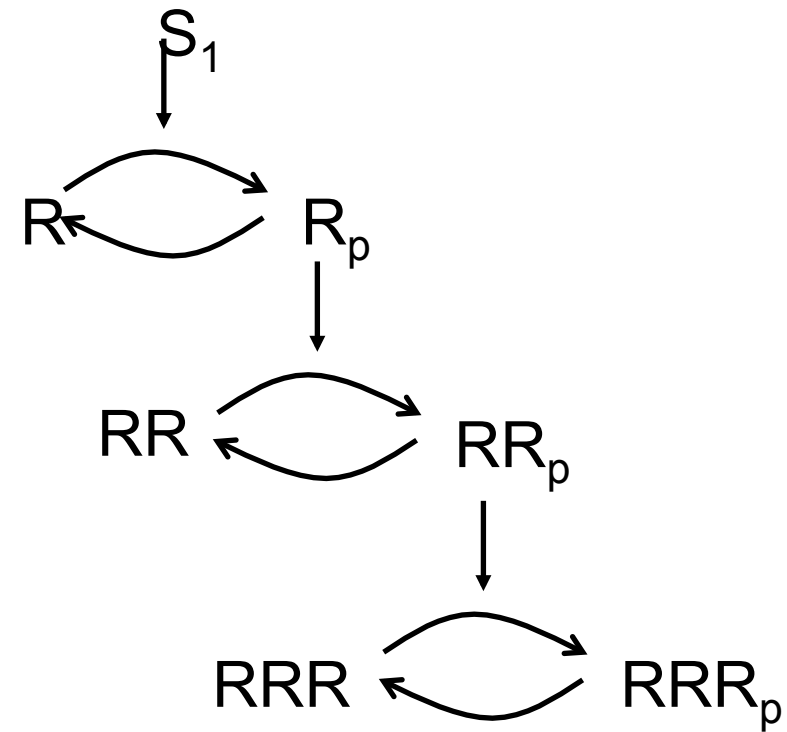


Phosphorylation cascade: 3-stage, Michaelis-Menten

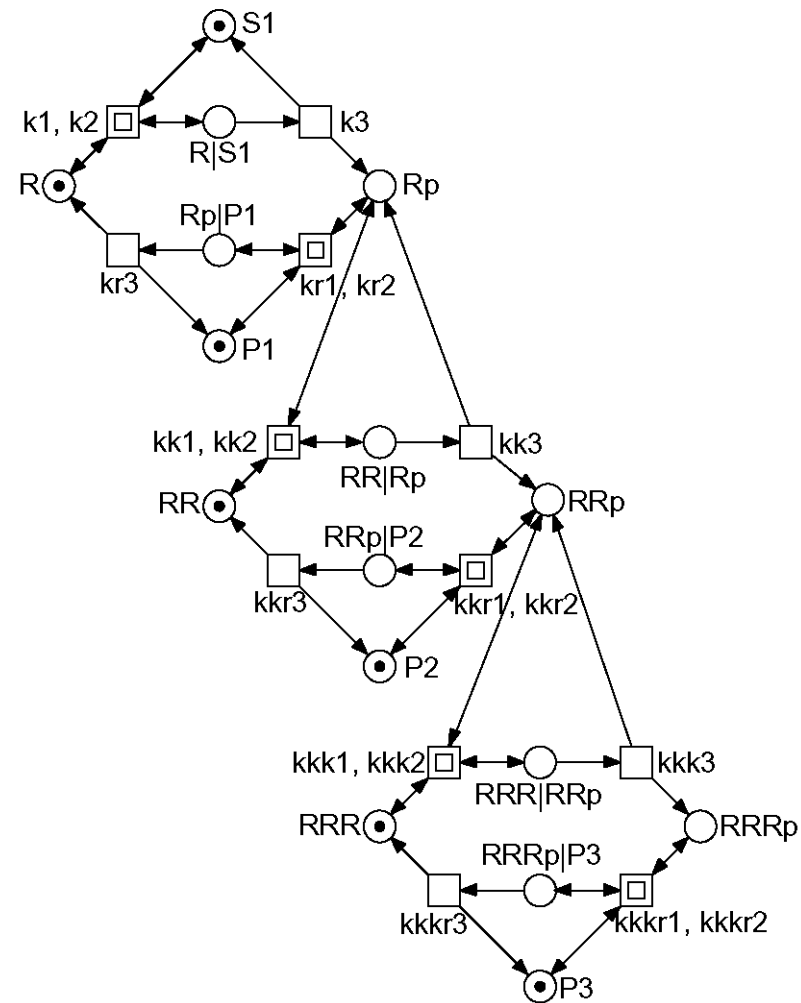
$$\frac{dR_p}{dt} = \frac{k_3 \times S_1 \times R}{K_{m1} + R} - \frac{k_3' \times R_p}{K_{m2} + R_p}$$

$$\frac{dRR_p}{dt} = \frac{kk_3 \times R_p \times RR}{KK_{m1} + RR} - \frac{kk_3' \times RR_p}{KK_{m2} + RR_p}$$

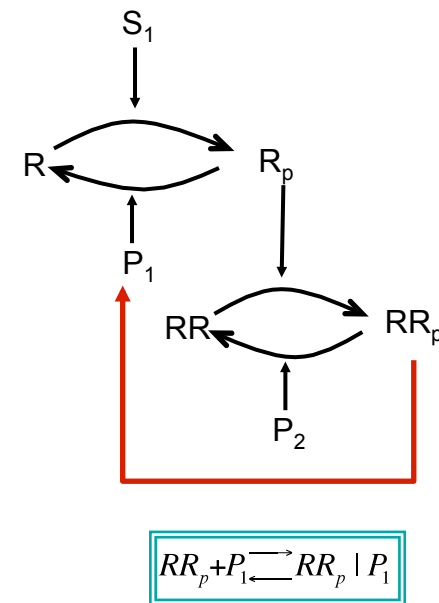
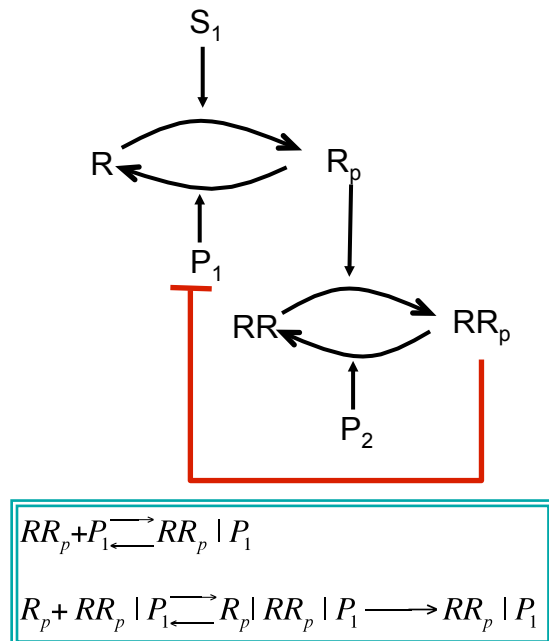
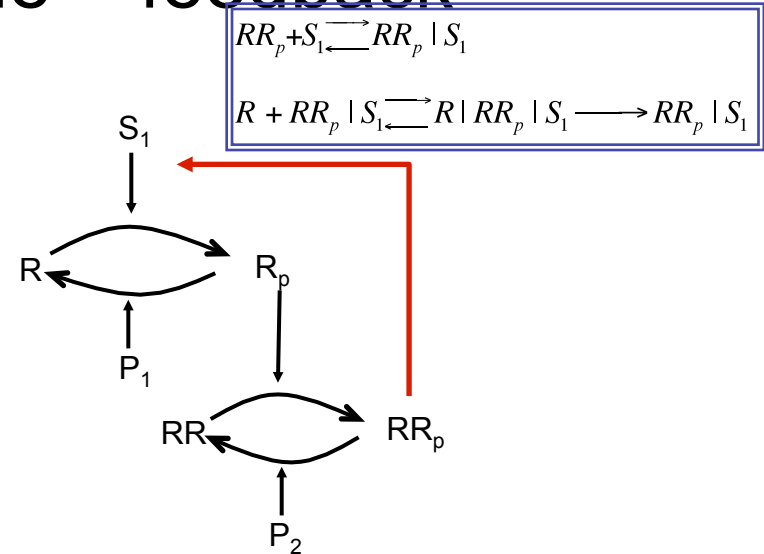
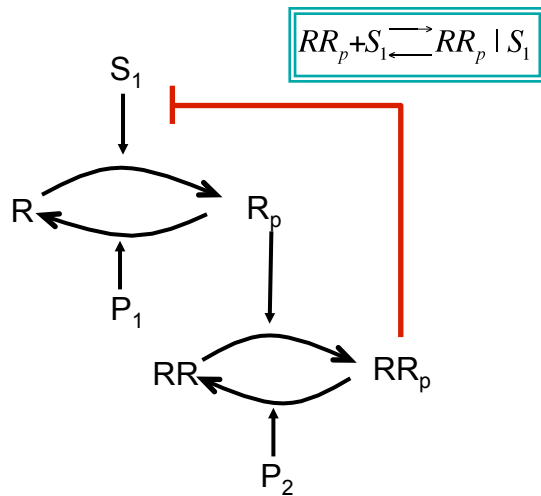
$$\frac{dRRR_p}{dt} = \frac{kkk_3 \times RR_p \times RRR}{KKK_{m1} + RRR} - \frac{kkk_3' \times RRR_p}{KKK_{m2} + RRR_p}$$



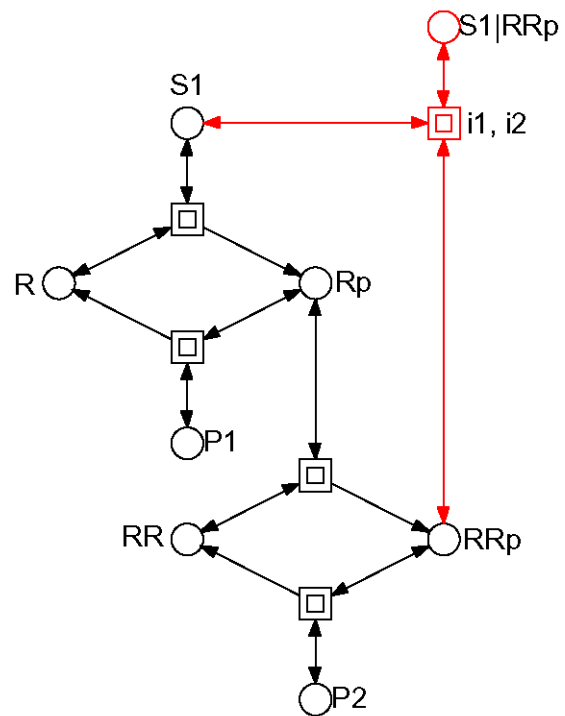
3-stage



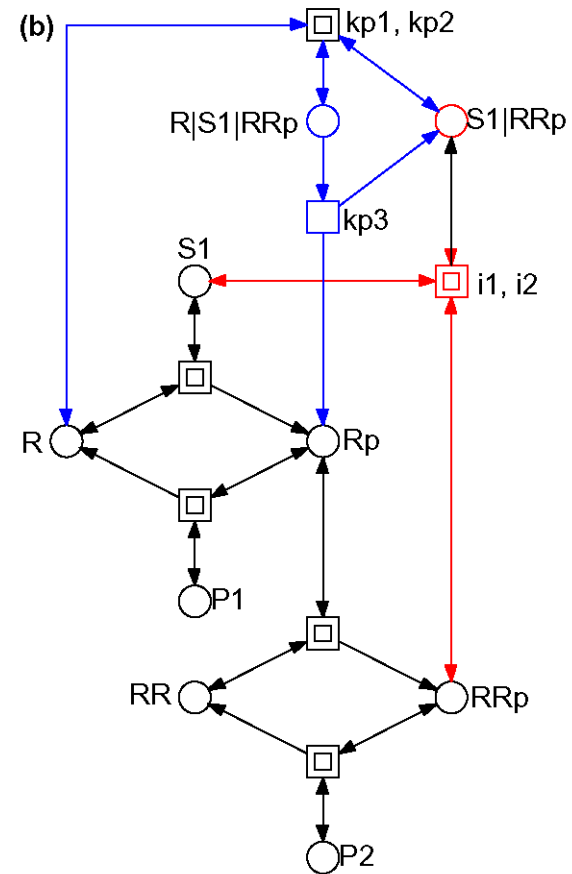
Phosphorylation cascade + feedback



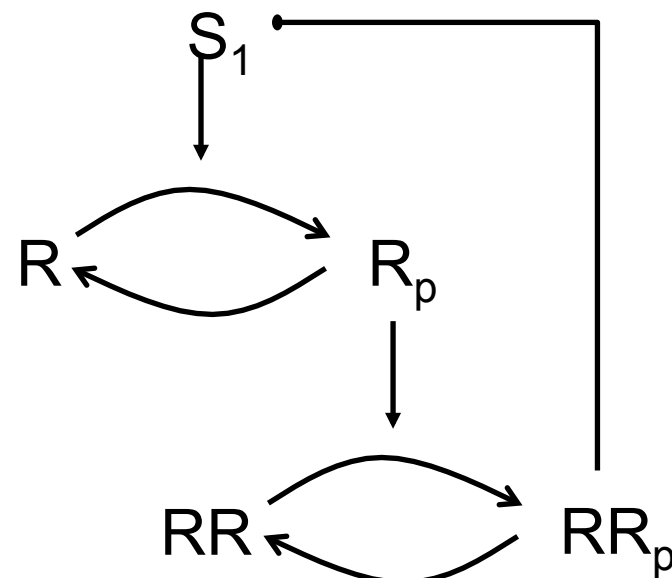
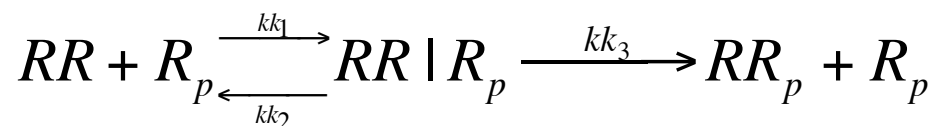
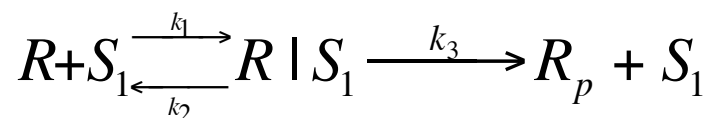
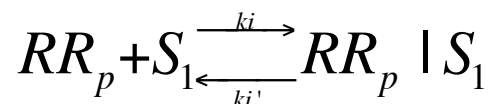
(a)



(b)



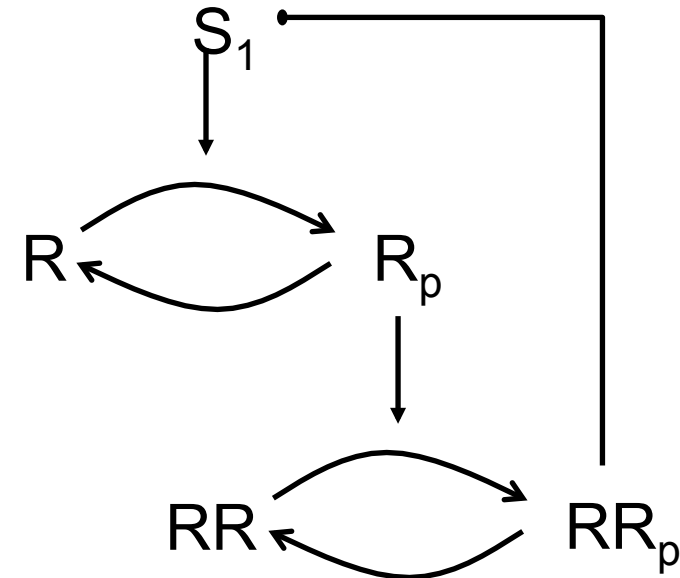
Phosphorylation cascade + negative feedback: 2-stage, Mass Action model 1



Phosphorylation cascade + negative feedback: 2-stage, Michaelis-Menten

$$\frac{dR_p}{dt} = \frac{k_3 \times S_1 \times R}{K_{m1} \times \left(1 + \frac{RR_p}{K_i}\right) + R} - \frac{k_3' \times R_p}{K_{m2} + R_p}$$

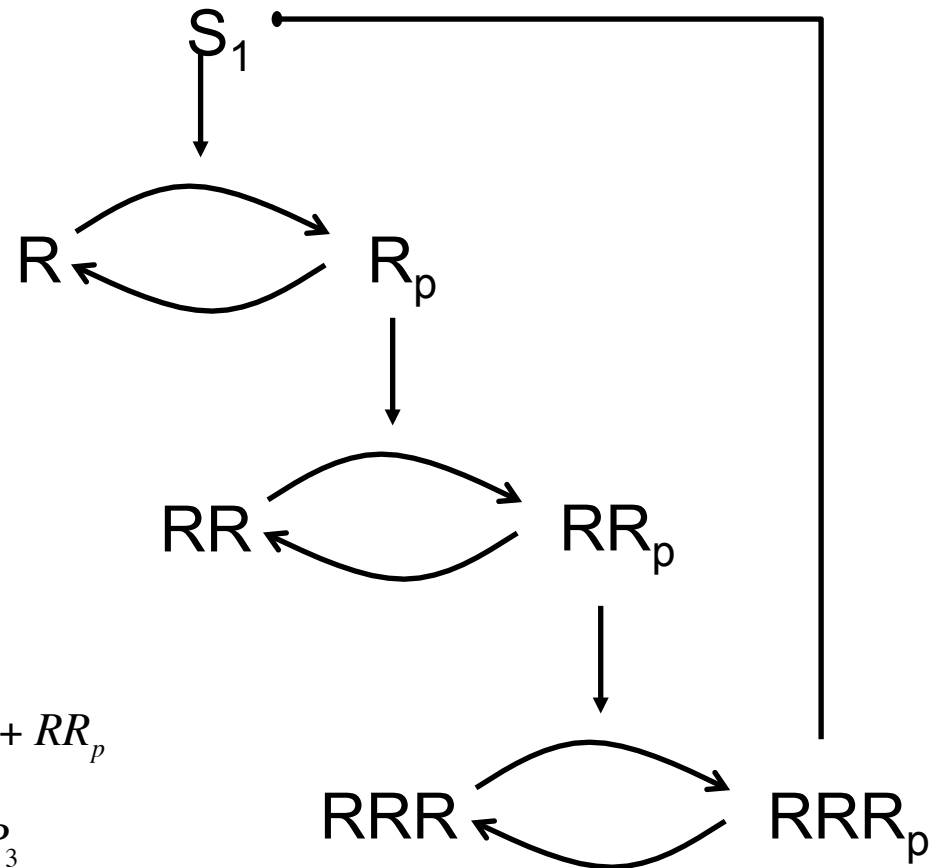
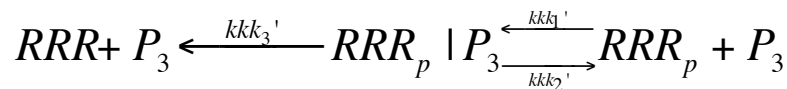
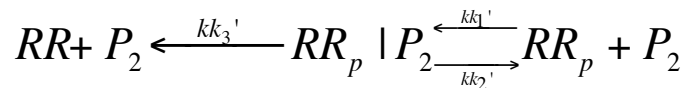
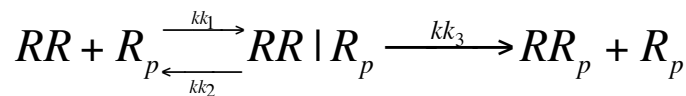
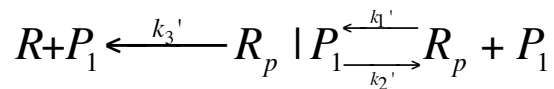
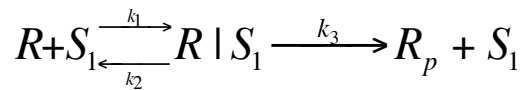
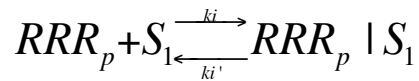
$$\frac{dRR_p}{dt} = \frac{kk_3 \times R_p \times RR}{KK_{m1} + RR} - \frac{kk_3' \times RR_p}{KK_{m2} + RR_p}$$



- Using Competitive Inhibition
- K_i is the dissociation constant for the SI complex

$$V = V_{\max} \times \frac{[S]}{[S] + K_m \times \left(1 + \frac{[I]}{[K_i]}\right)}$$

Phosphorylation cascade + negative feedback: 3-stage, Mass Action, model 1



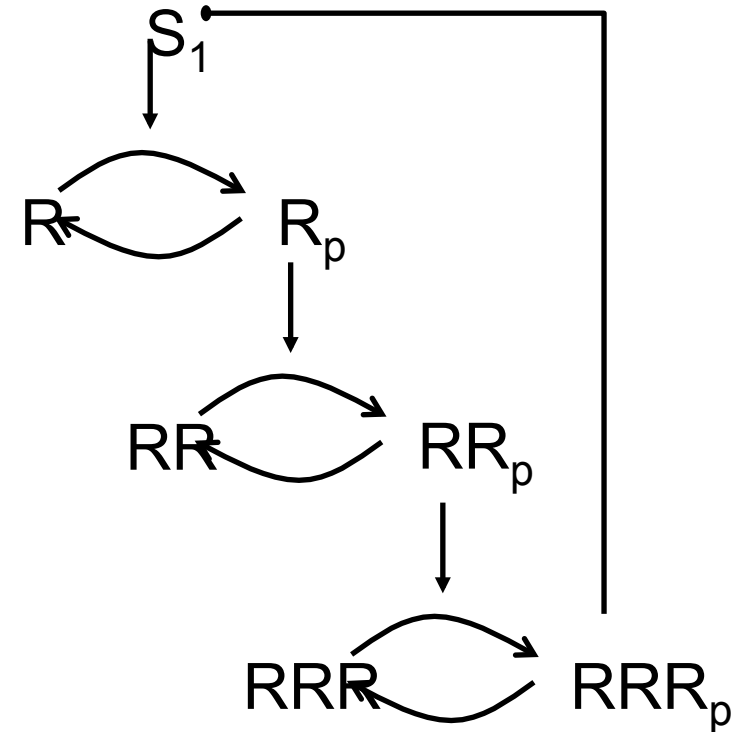
Phosphorylation cascade + negative feedback: 3-stage, Michaelis-Menten

$$\frac{dR_p}{dt} = \frac{k_3 \times S_1 \times R}{K_{m1} \times \left(1 + \frac{RRR_p}{K_i}\right) + R} - \frac{k_3' \times R_p}{K_{m2} + R_p}$$

$$\frac{dRR_p}{dt} = \frac{kk_3 \times R_p \times RR}{KK_{m1} + RR} - \frac{kk_3' \times RR_p}{KK_{m2} + RR_p}$$

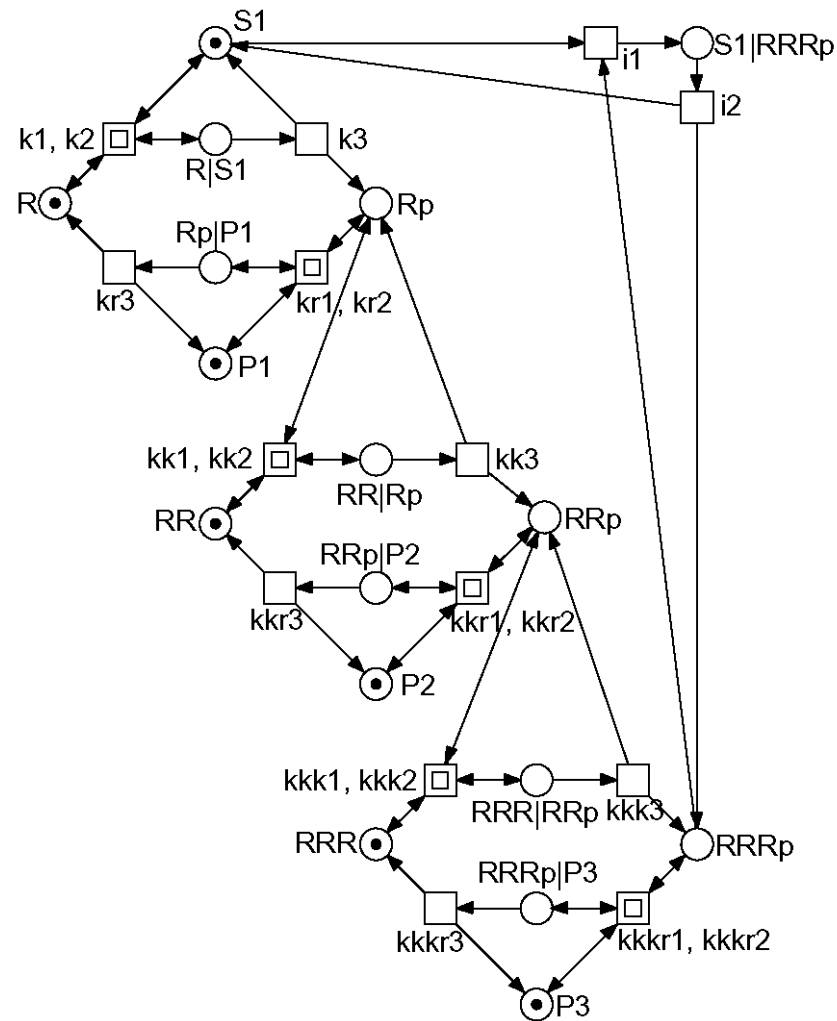
$$\frac{dRRR_p}{dt} = \frac{kkk_3 \times RR_p \times RRR}{KKK_{m1} + RRR} - \frac{kkk_3' \times RRR_p}{KKK_{m2} + RRR_p}$$

- Using Competitive Inhibition
- K_i is the dissociation constant for the SI complex

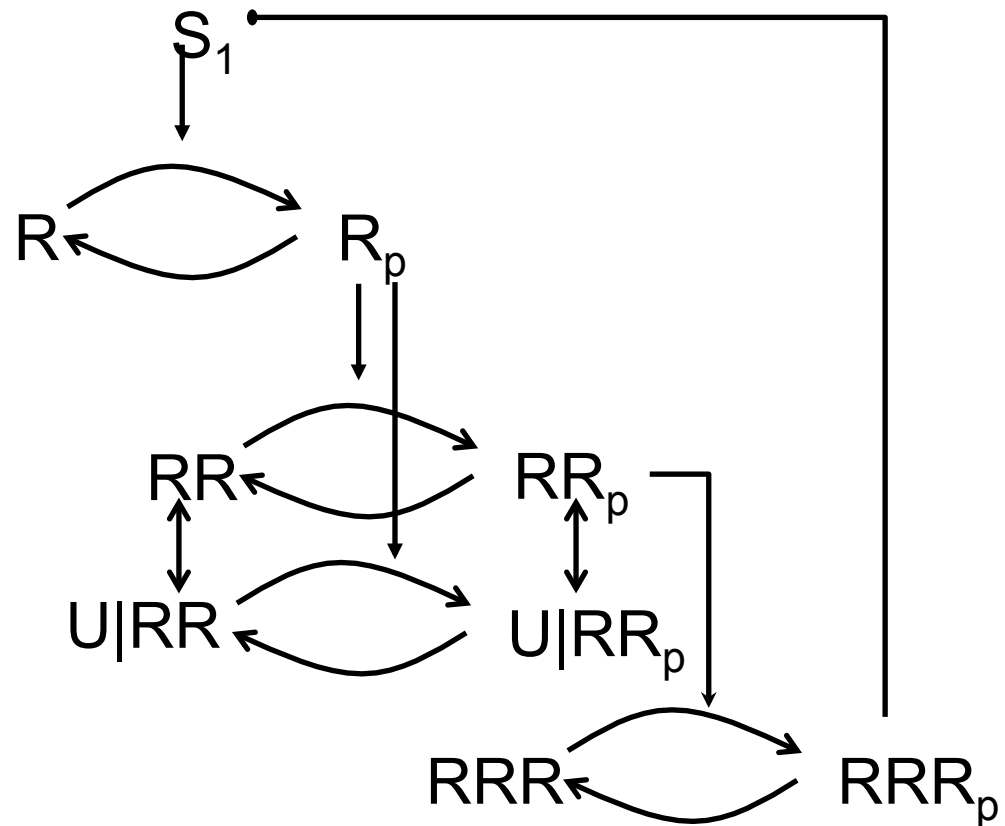
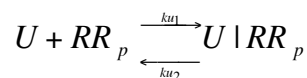
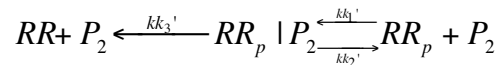
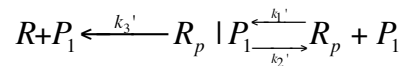
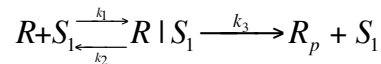
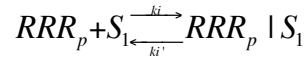


$$V = V_{\max} \times \frac{[S]}{[S] + K_m \times \left(1 + \frac{[I]}{[K_i]}\right)}$$

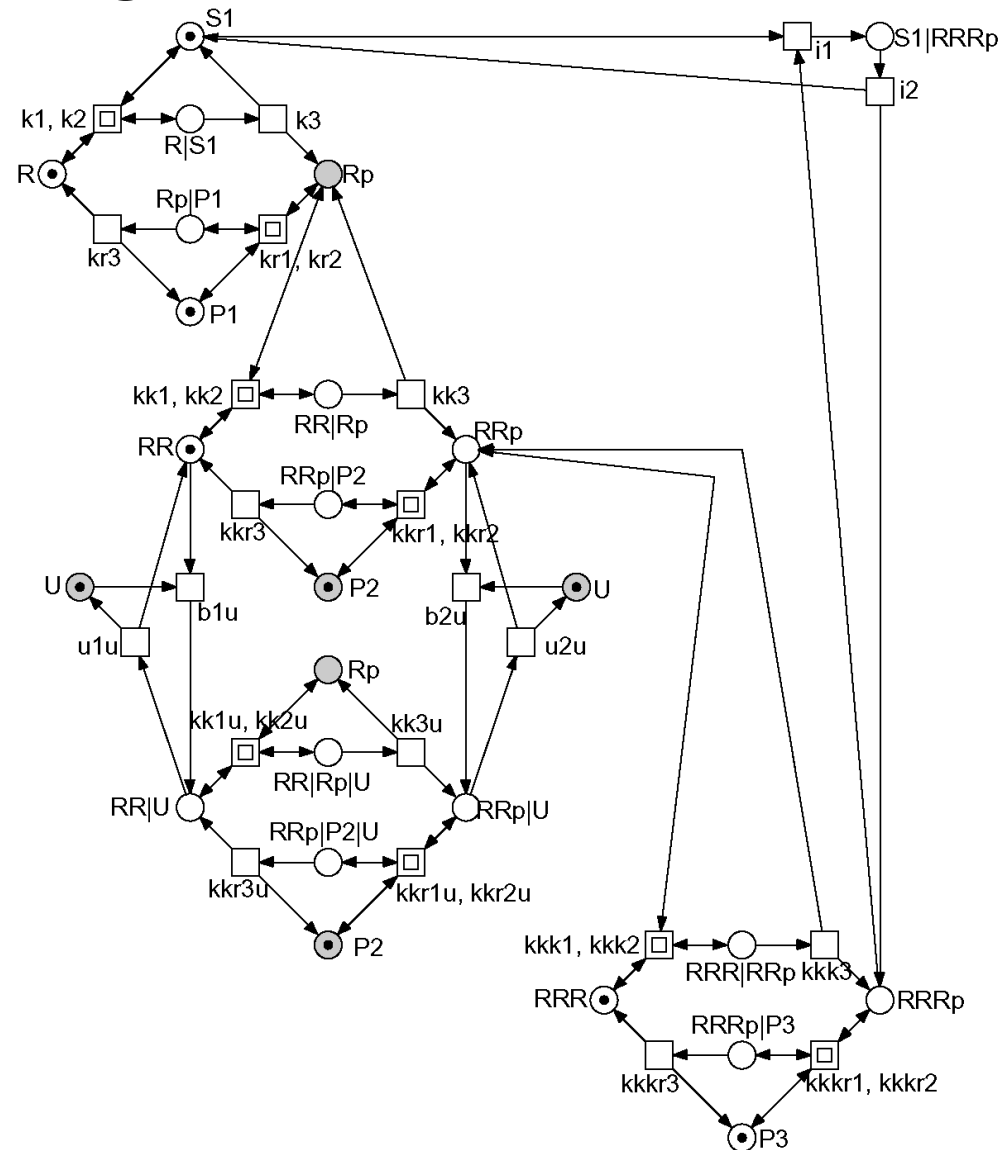
3-stage, negative feedback



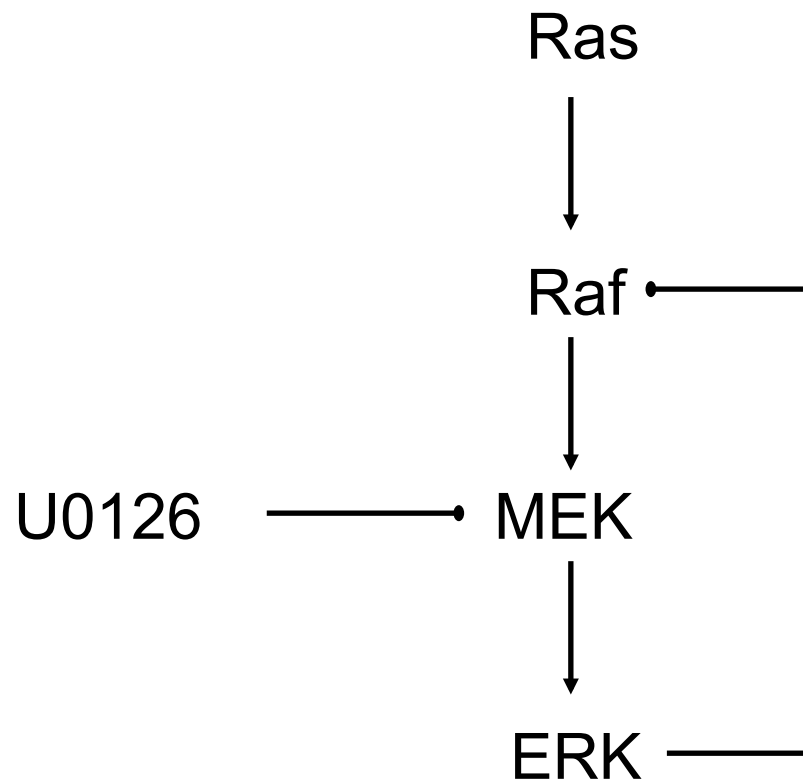
Phosphorylation cascade + negative feedback: 3-stage, Inhibitor on 2nd stage, Mass Action



3-stage, negative feedback + inhibitor



‘Real cascade & feedback’

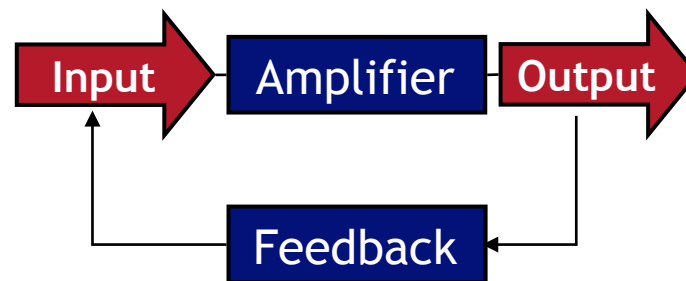


Is the ERK pathway a negative feedback amplifier?

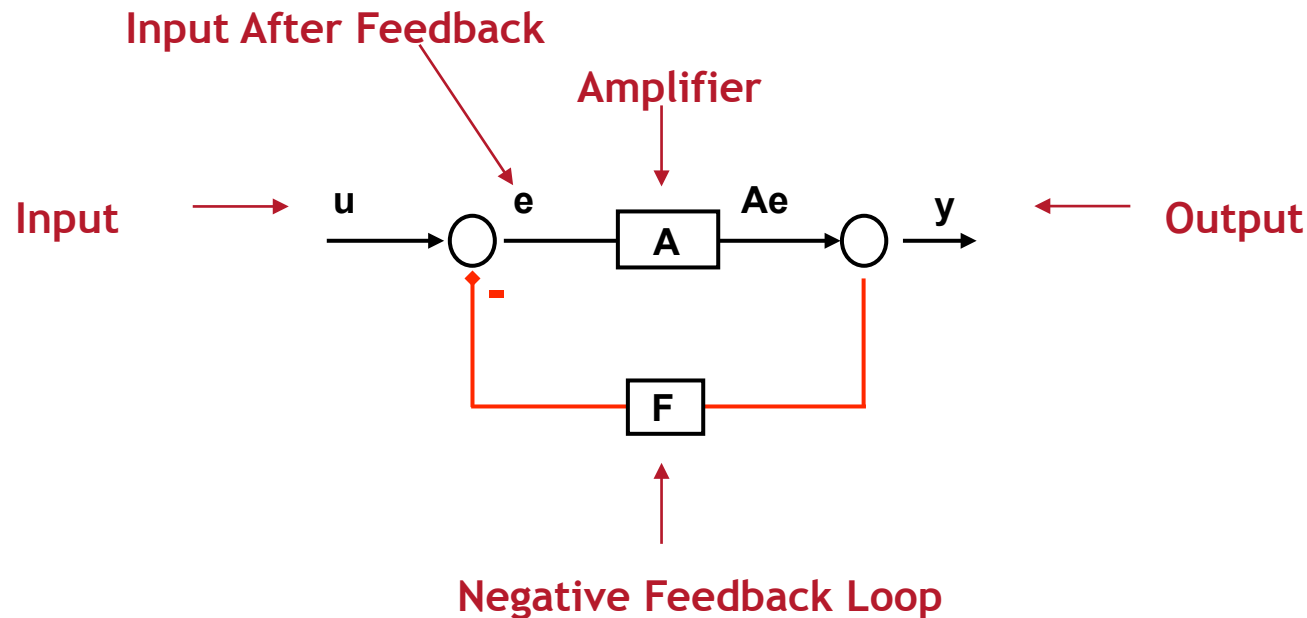
Sauro HM, Kholodenko BN.
Quantitative analysis of signaling networks.
Prog Biophys Mol Biol. 2004 Sep;86(1):5-43.

Negative Feedback Amplifier

- A negative feedback amplifier stems from the field of electronics and consists of an amplifier with a negative feedback loop from the output of the amplifier to its input.
- The negative feedback loop results in a system that is much more robust to disturbances in the amplifier.
- The negative feedback amplifier was invented in 1927 by Harold Black of Western Electric and was originally used for reducing distortion in long distance telephone lines.
- The negative feedback amplifier is now a key electrical component used in a wide variety of applications



Negative Feedback Amplifier



Steady State Equation

$$y = \frac{Au}{1 + AF}$$

Biological Applications

$$y = Ae$$

$$e = u - Fy$$

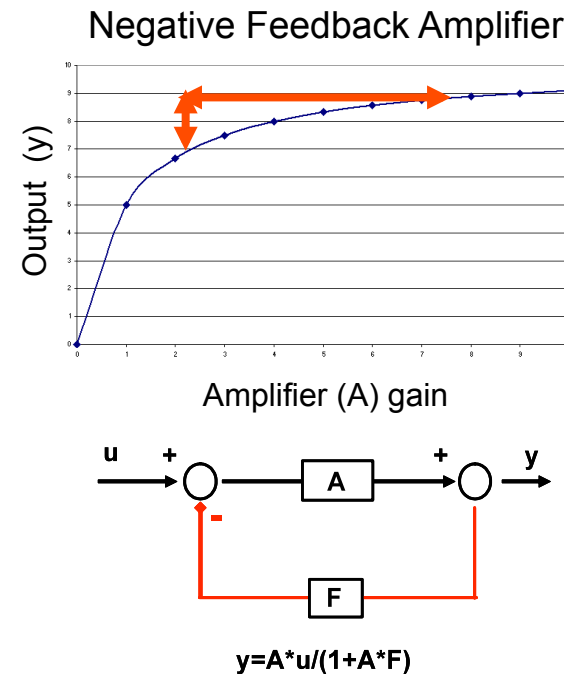
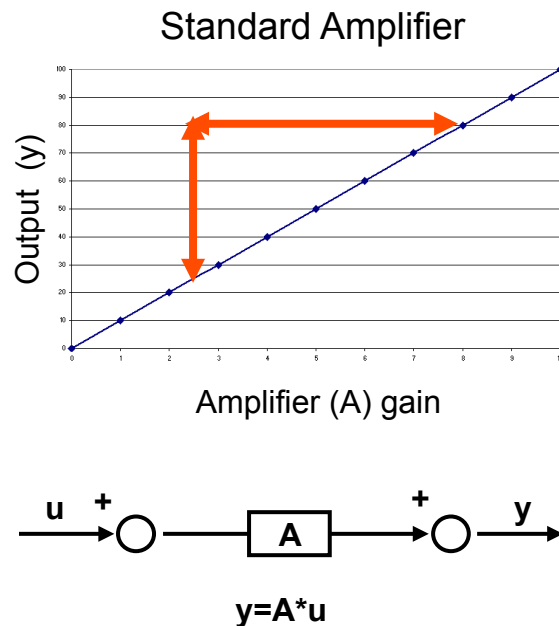
$$y = A(u - Fy)$$

$$y = Au - AFy$$

$$y + AFy = Au$$

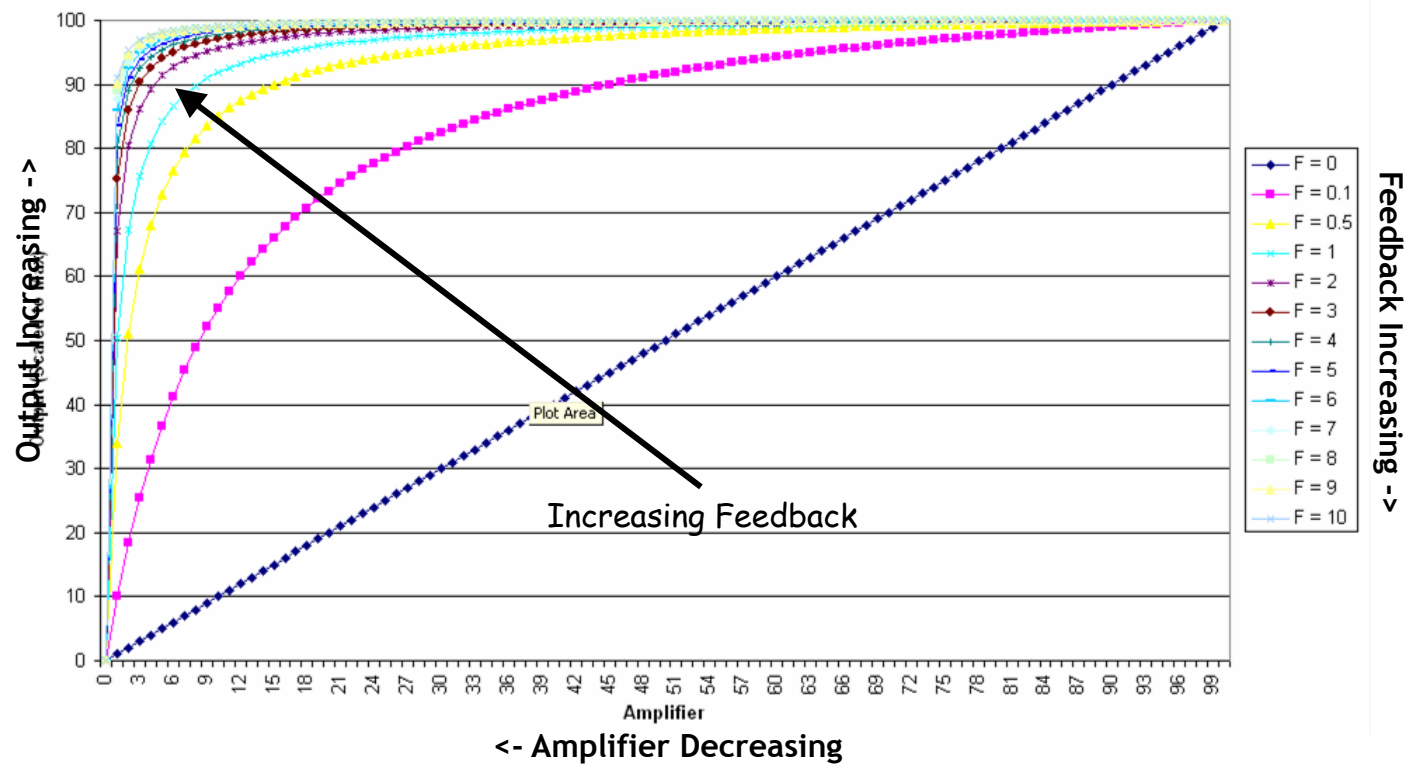
$$y(1 + AF) = Au$$

The negative feedback imparts signalling robustness



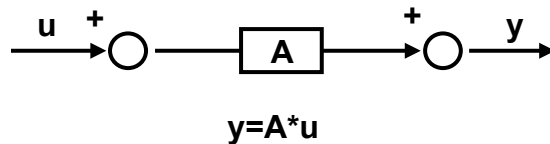
A large change in amplifier gain leads to a small change in output (y)

Feedback

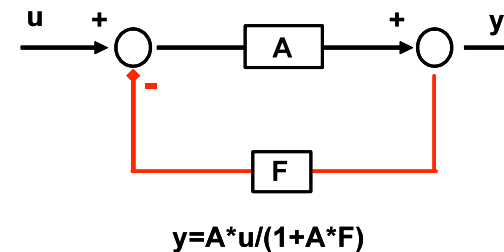


The negative feedback imparts signalling robustness

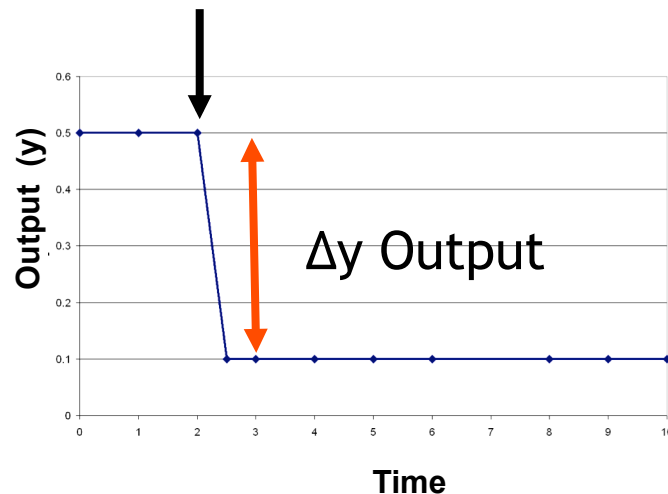
Standard Amplifier



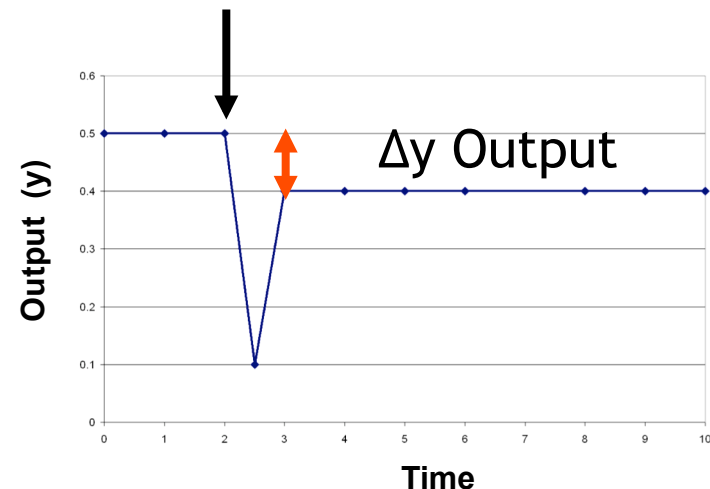
Negative Feedback Amplifier



Sudden drop in Amplifier (A) gain

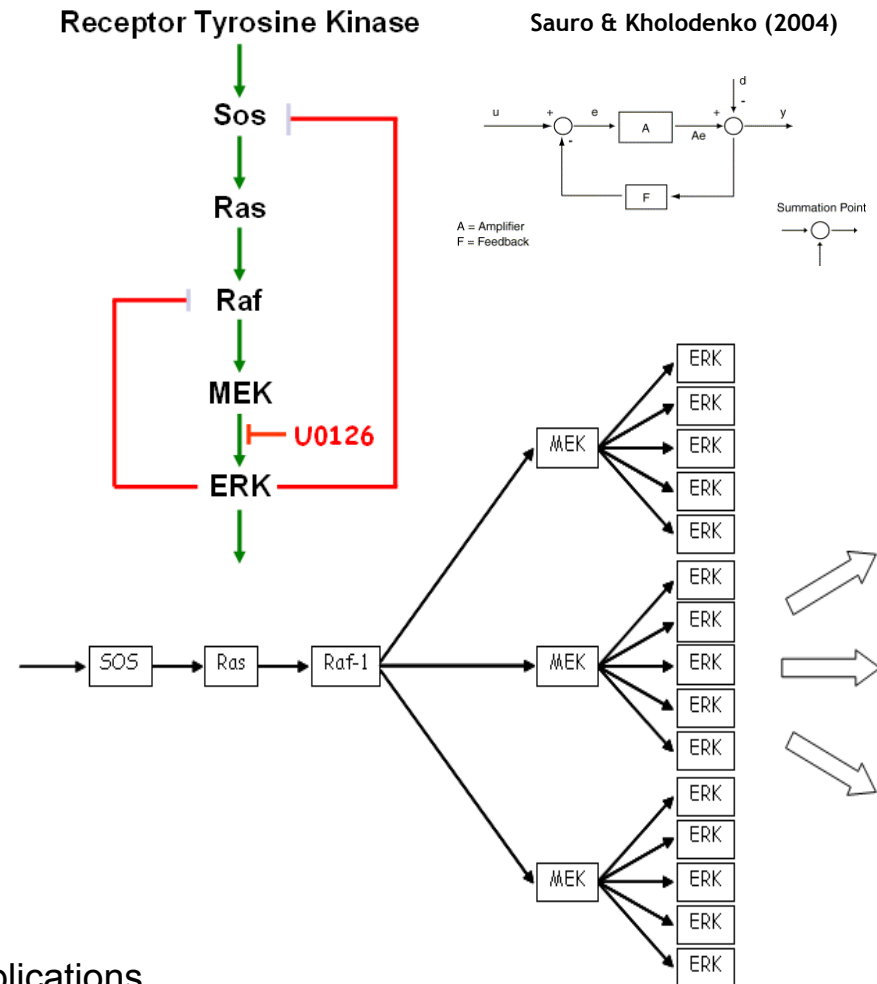


Sudden drop in Amplifier (A) gain

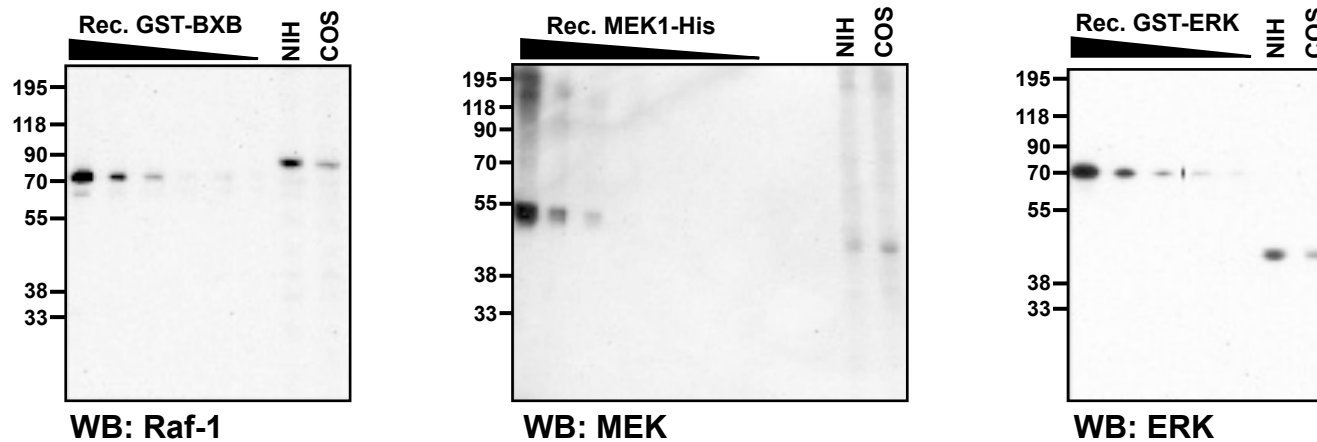


Application to Biology

- The ERK cascade is a well known biological amplifier and contains numerous negative feedback loops.
- At first sight, it has the correct structure to be a negative feedback amplifier.
- If the ERK cascade is a negative feedback amplifier it should be robust to disturbances within the cascade.
- From a biological point of view, these disturbances could be caused by drugs, such as U0126, aimed at decreasing the activity of the ERK cascade.
- This suggests that these drugs will be relatively ineffective.
- In fact, current drugs aimed at decreasing the activity of the MAPK pathway have proved less efficient in *in vivo* applications than anticipated from *in vitro* inhibition assays.

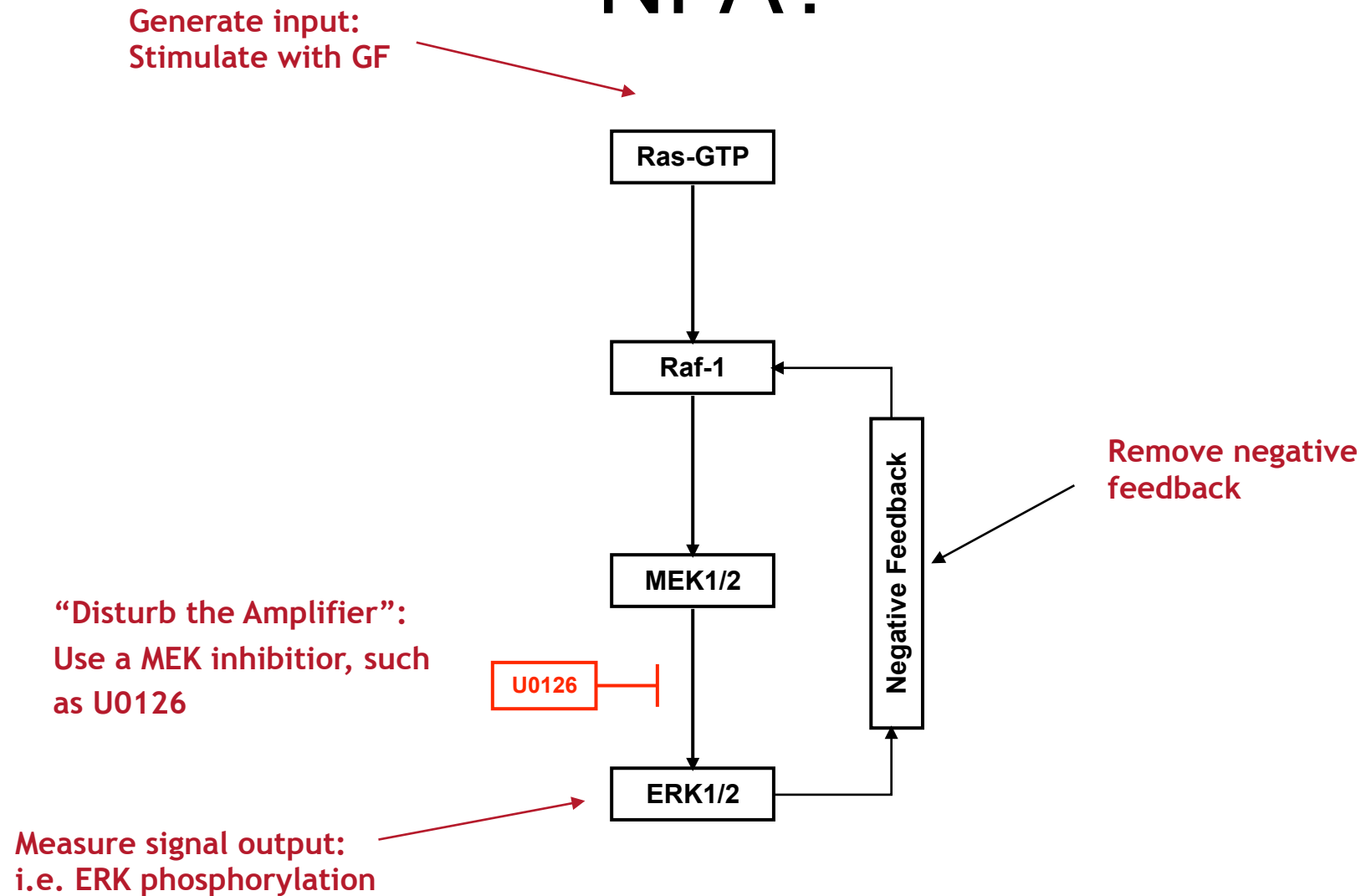


Raf/MEK/ERK amplifies the signal



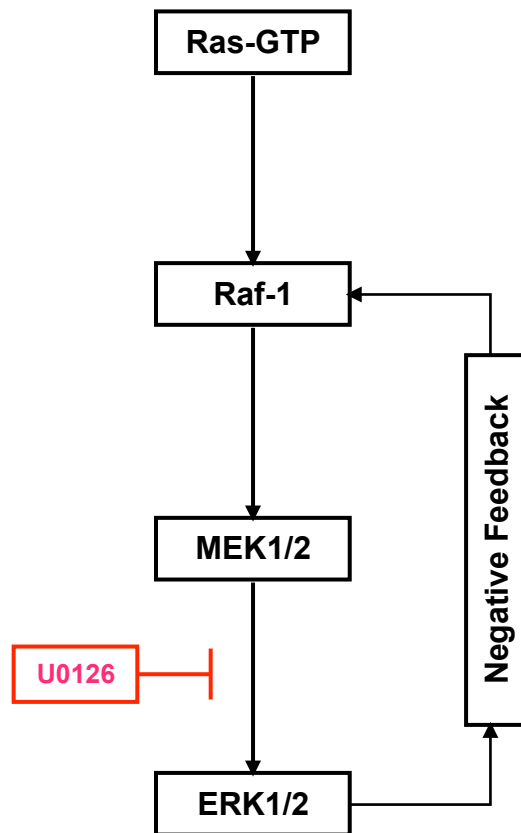
Cell line	Raf-1	MEK	ERK	Concentration per cell
COS1	3.6	10.6	21.2	femtomol
	1	2.9	5.9	ratio
NIH 3T3	10.9	7.1	98	femtomol
	1	0.7	9	ratio

How to test if the ERK pathway is a NFA?

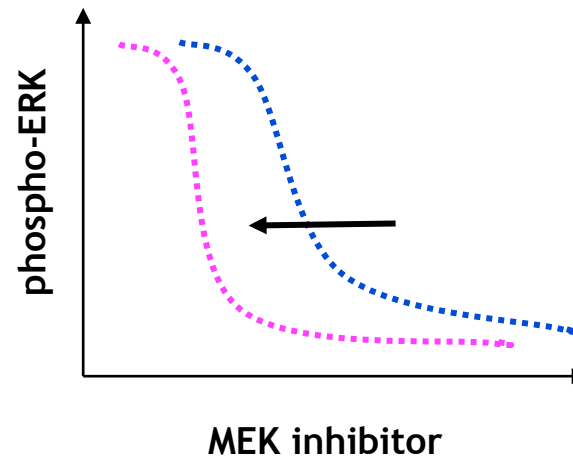


Hypothesis: Braking the feedback should sensitise the ERK pathway to MEK-inhibitor

Feedback intact

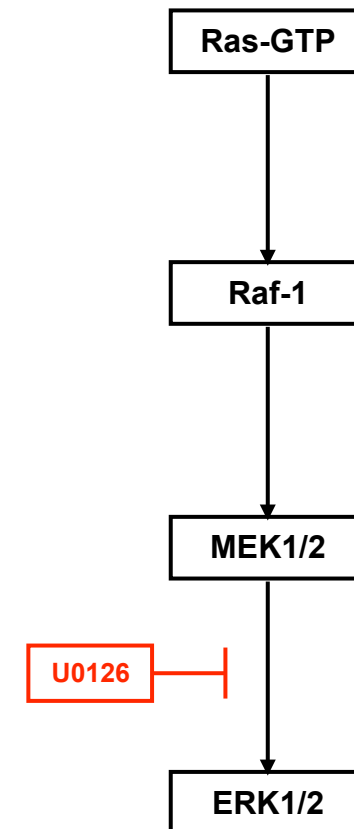


drg@brc.dcs.gla.ac.uk



Biological Applications

Feedback removed



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How to test if the ERK pathway is a NFA?

Strategy



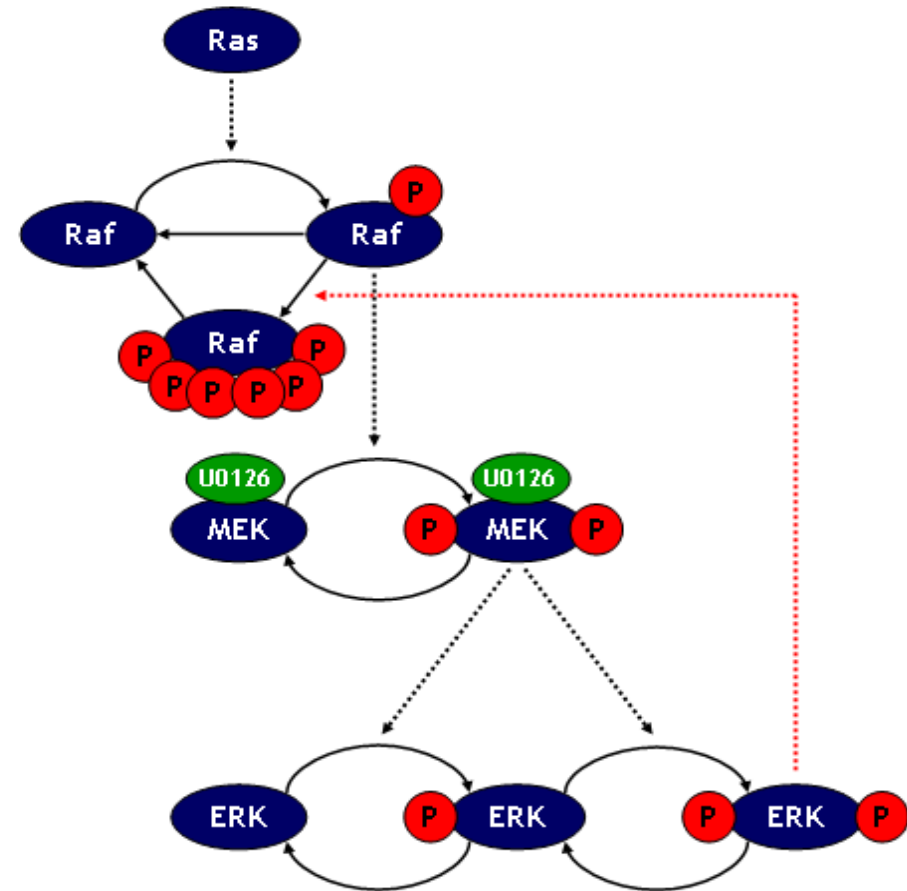
In vivo system that allows us to compare feedback broken to feedback intact model.

Computational Model of ERK pathway with/without feedback

Computational Modeling 1:

Build the model

- Non-linear ordinary differential equations (ODE's).
- ODE's were solved using Math Lab and Gepasi.
- Models are based on the Schoeberl et al. (2002) model
- Mass Action Kinetics instead of Michaelis Menten
- Kinetic parameters are from literature, previous models and "guesstimates"



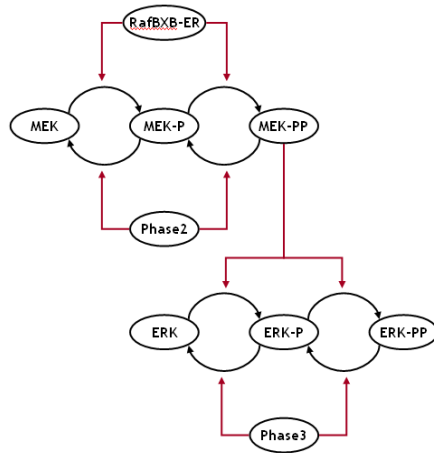
Amplifier / negative feedback

- Model amplifier strength by
 - Adding inhibitor to 2nd stage
 - Modifying k_{k3} , k_{kk3} [i.e. modifying rate of production of RRp , $RRRp$]
 - Add/remove cascade elements
- Then plot amp strength versus output, e.g. $[U]$ vs $[RRRp]$
- ?Model feedback strength by
 - Leaving out feedback loop
 - varying k_i , and plot k_i vs $[RRRp]$
- Notes: avoid saturation; use signal in linear range; ? model degradation in S1 signal?

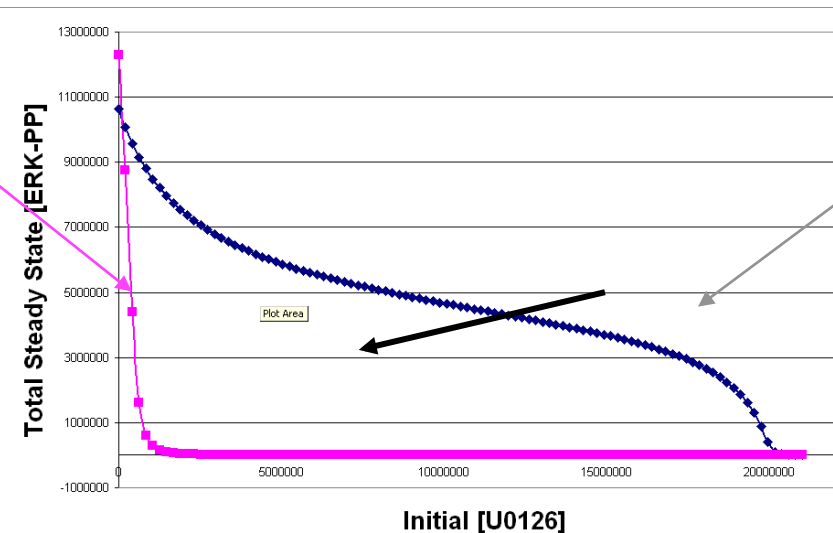
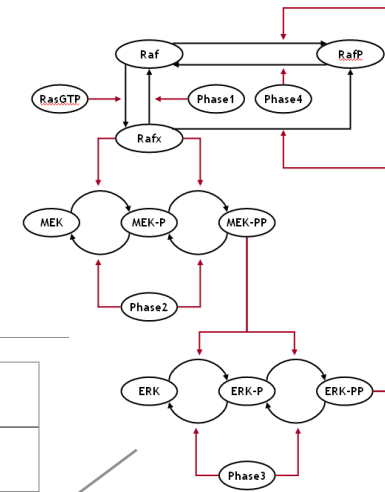
Computational Modeling 2:

Results

Feedback broken



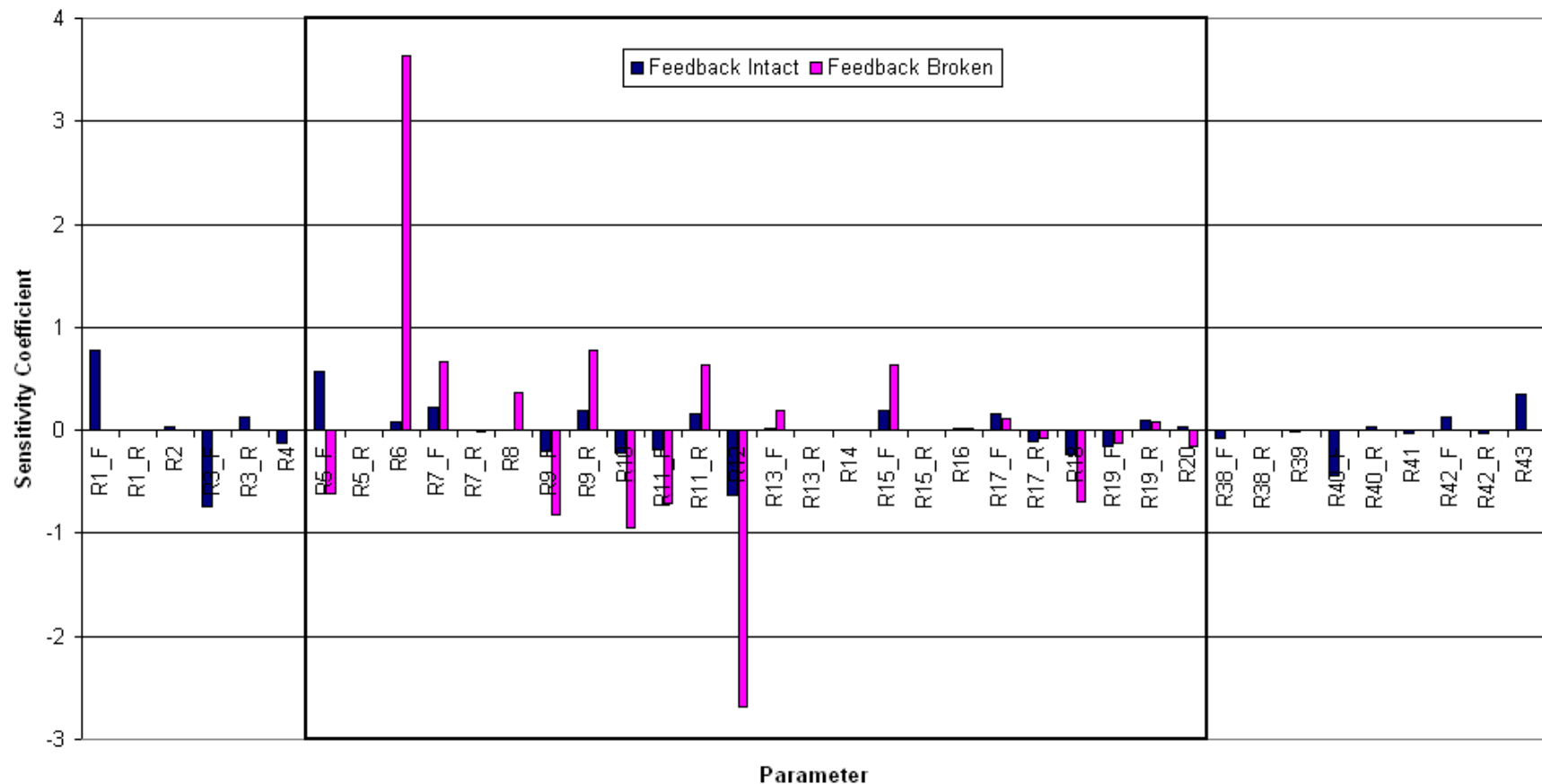
Feedback intact



Prediction: Braking the feedback modulates drug response

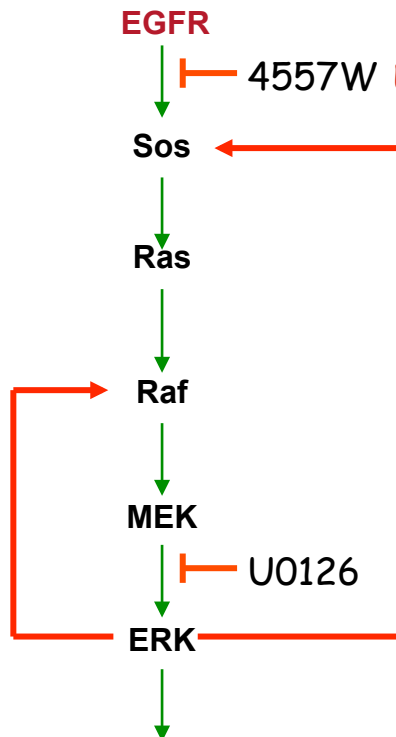
Computational Modeling 2:

Results
Sensitivity of kinetic parameters is decreased due to Negative Feedback

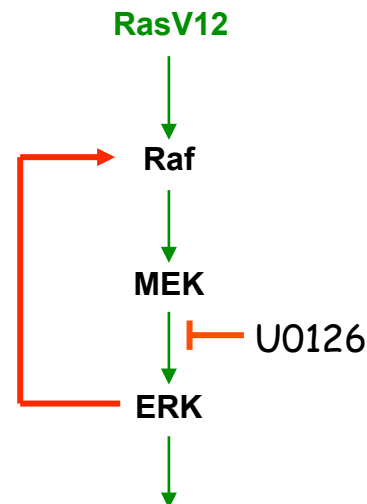


The experimental systems

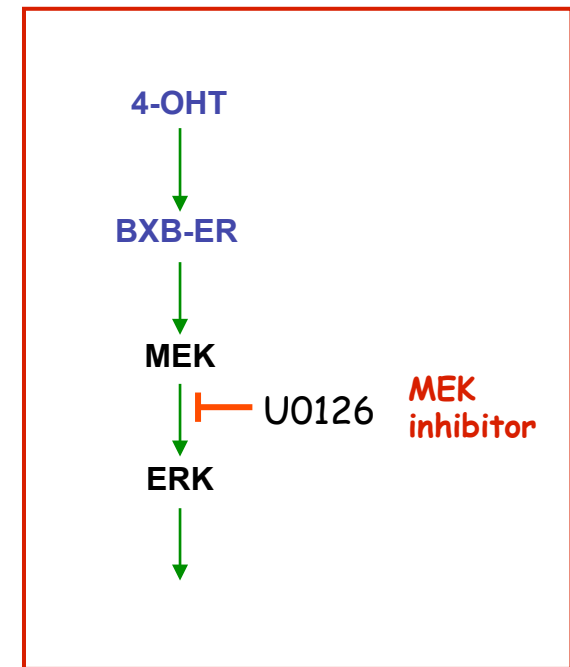
Negative feedback loops intact



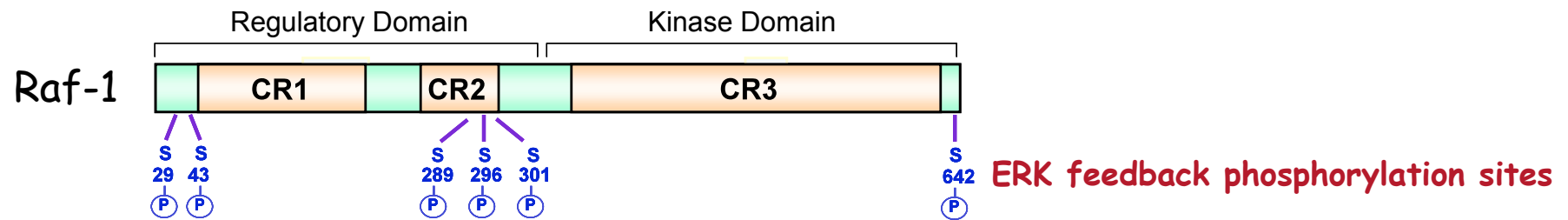
One feedback loop eliminated by constitutively active RasV12 mutant



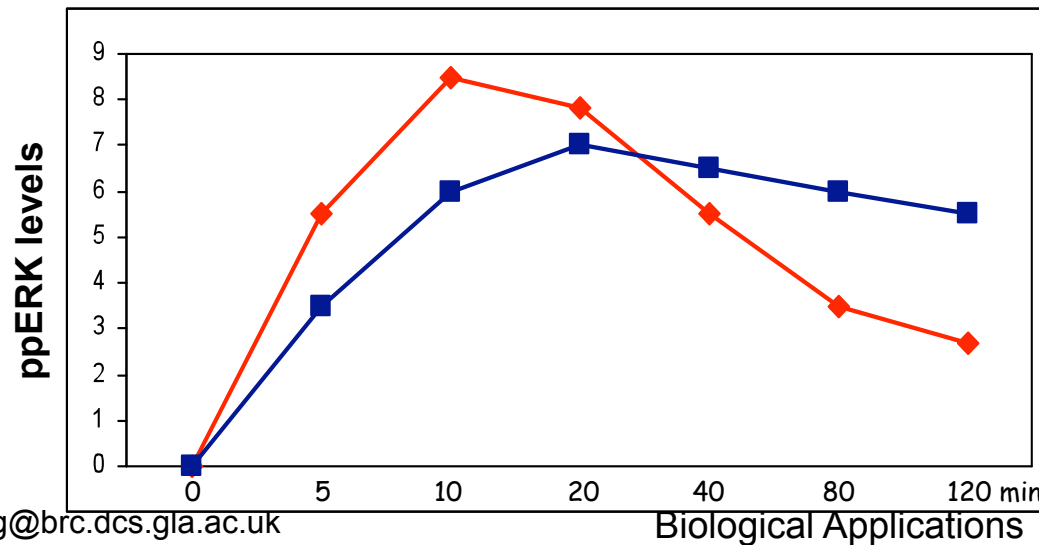
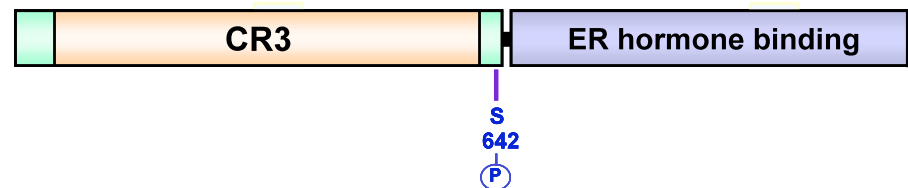
Both feedback loops eliminated by BXB-ER (4-OHT regulatable Raf-1 mutant)



Breaking the ERK feedback with BXB-ER



BXB-ER

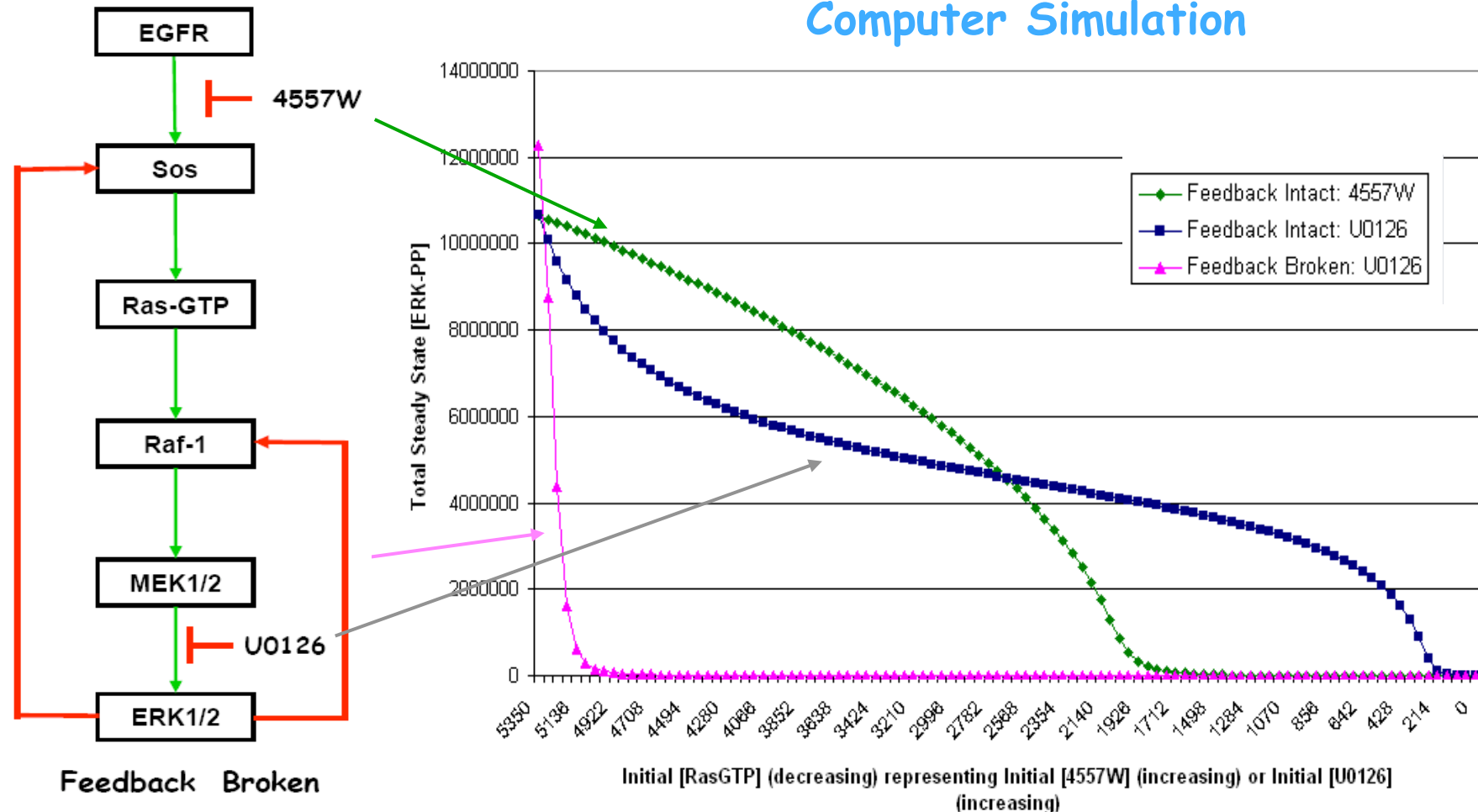


BXB-ER stimulated with 4-OHT
(4-Hydroxy Tamoxifen, a synthetic estrogen)

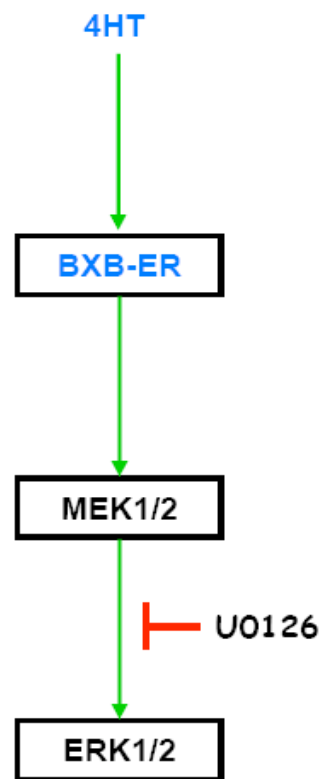
Raf-1 stimulated with EGF

Ablation of feedback by BXBER decreases robustness to MEK-inhibitor U0126

Computer Simulation

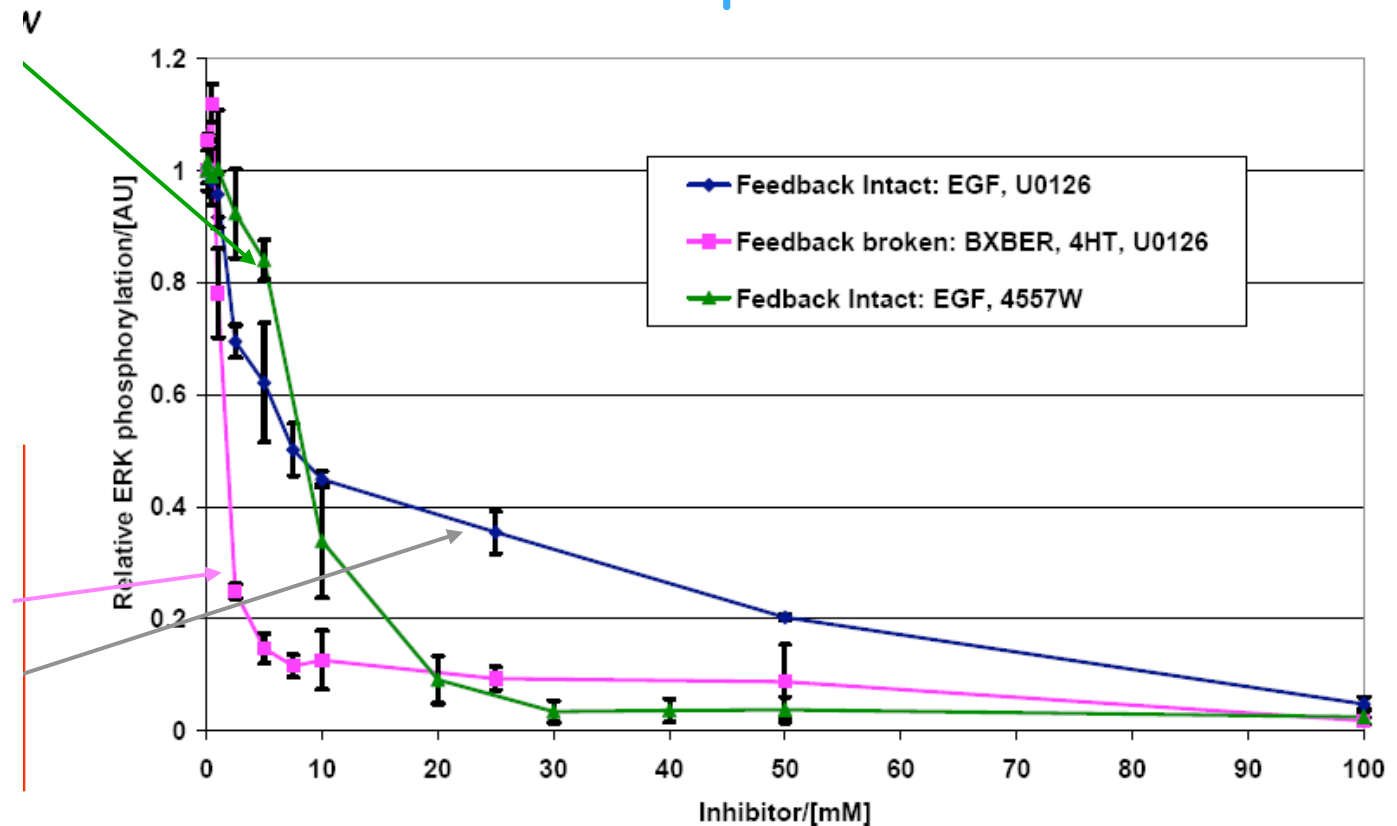


Ablation of feedback by BXBER decreases robustness to MEK-inhibitor U0126



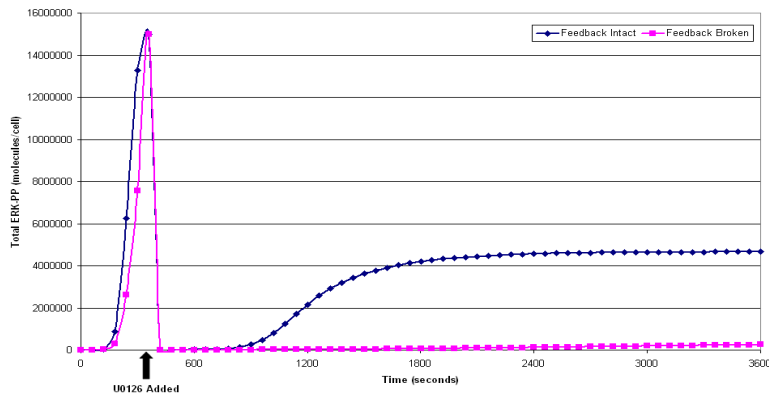
Feedback Broken

Experiment

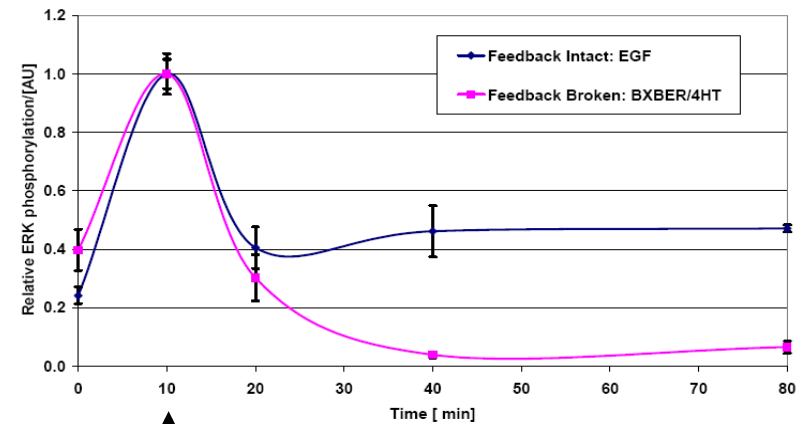


Signal recovery after MEK inhibition

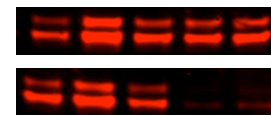
Simulation



Experiment



U0126 added

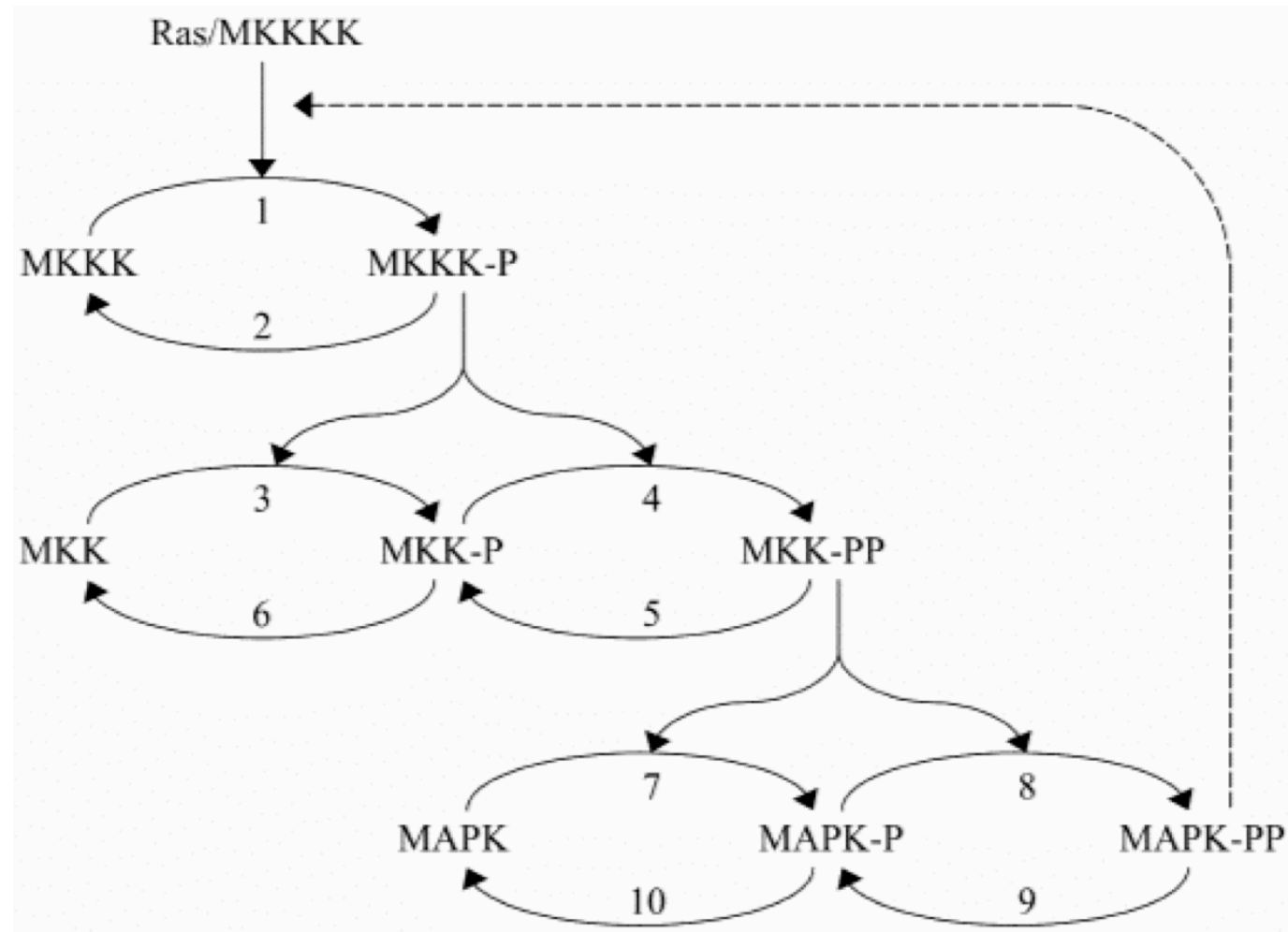


pERK1/2, +EGF

pERK1/2, + BXBER/4HT

0 10 20 40 80 min stimulation

Kholodenko – negative feedback oscillator



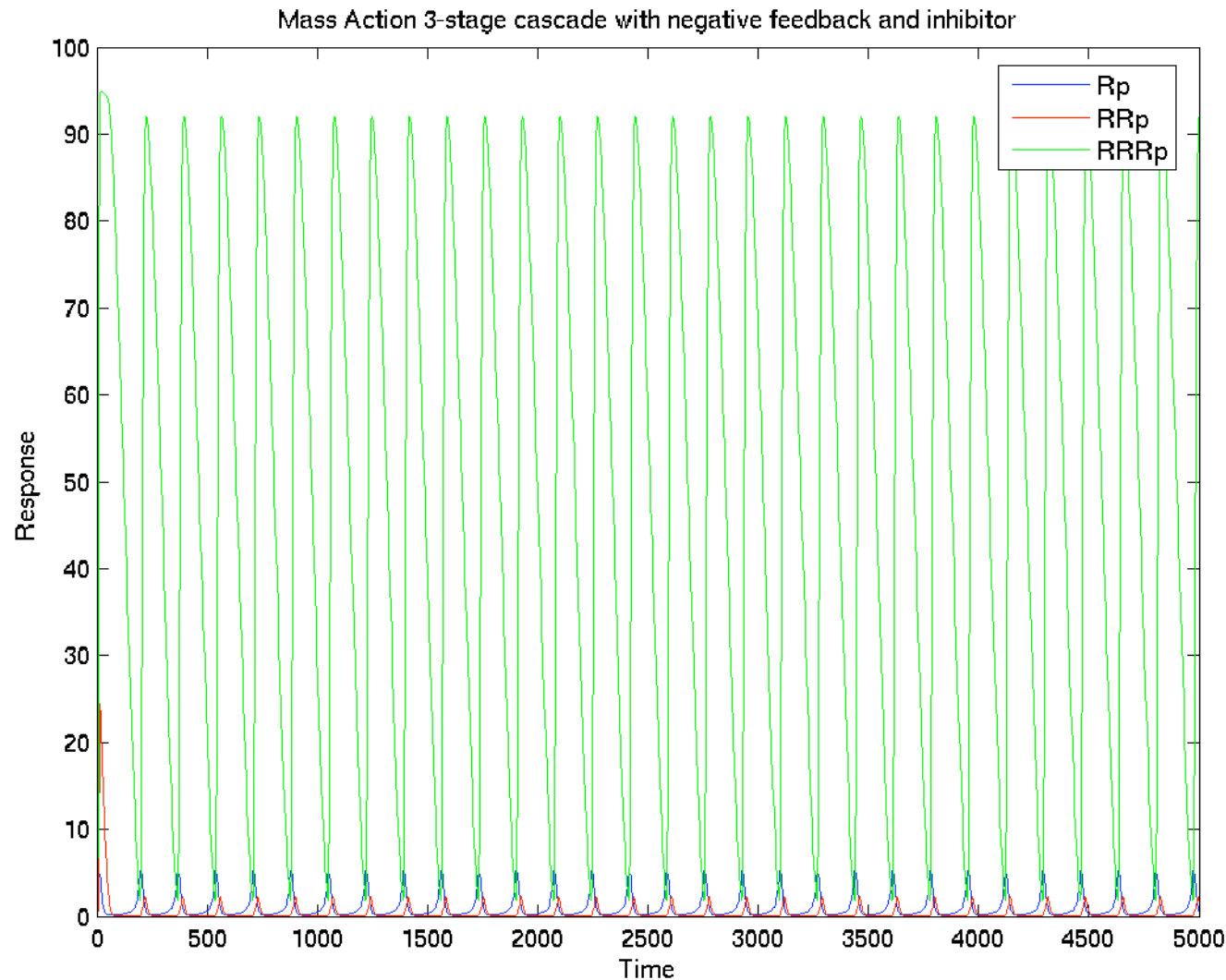
Negative feedback and ultrasensitivity can bring about oscillations in the mitogen-activated protein kinase cascades.
Kholodenko BN., Eur J Biochem 2000 Mar;267(6):1583-8

Oscillations! Phosphorylation cascade + negative feedback: 3-stage, Inhibitor on 2nd stage, Mass Action

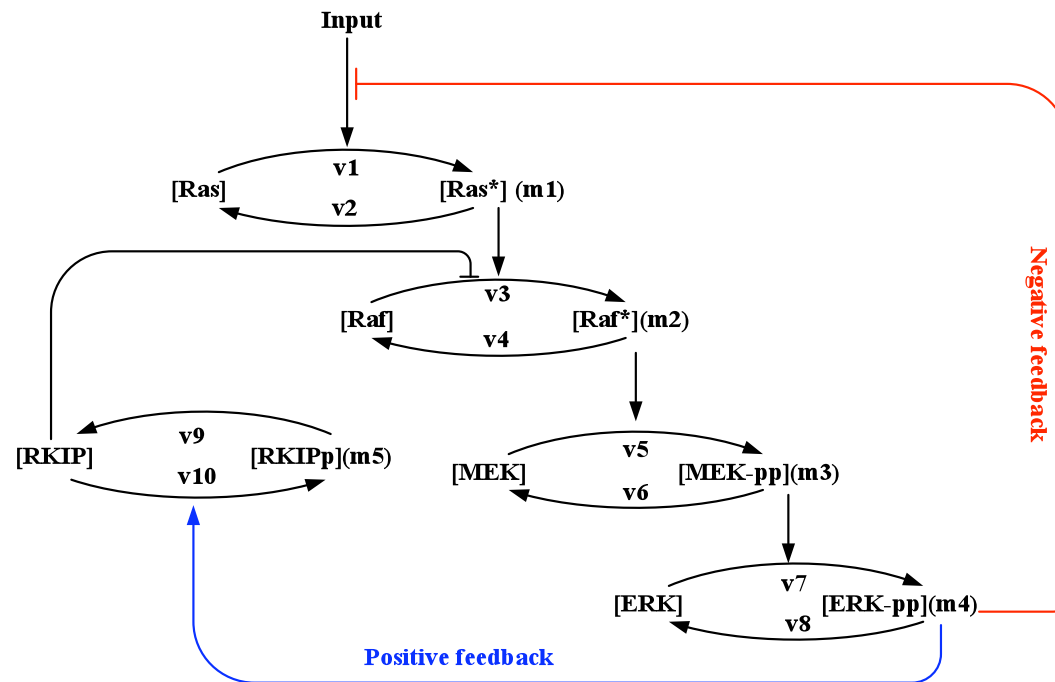
Conditions

$S1=3$

Inhibitor=0.5

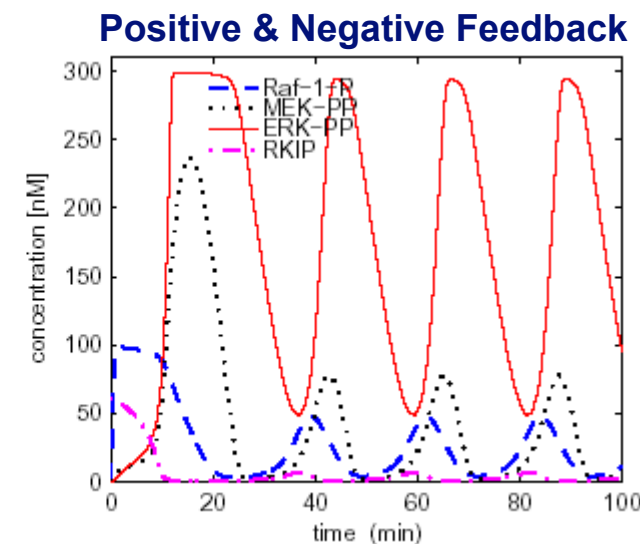
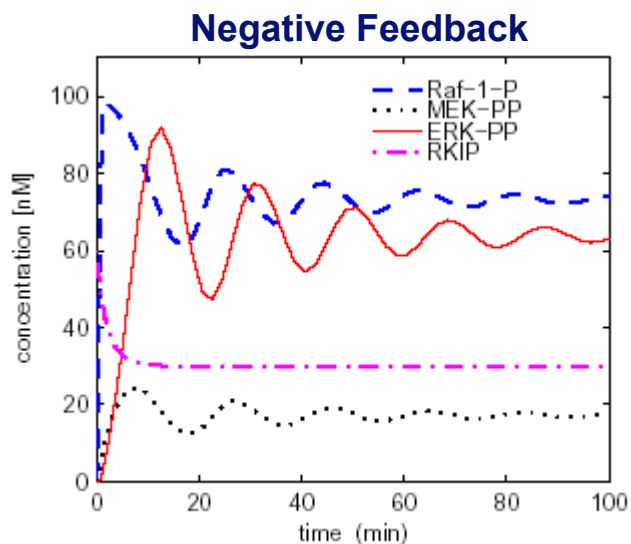
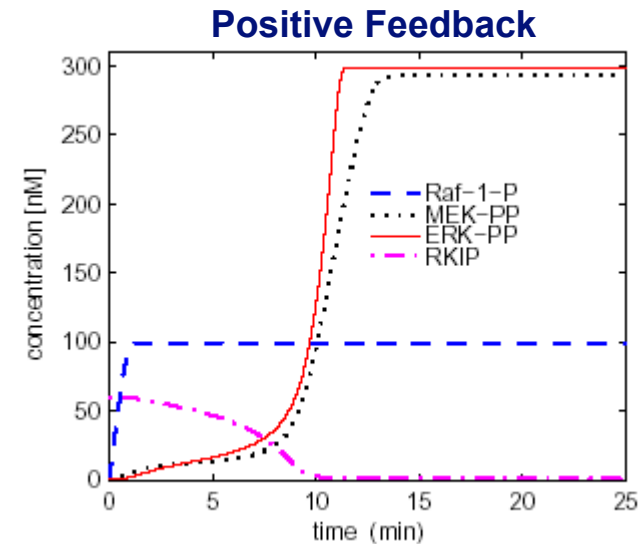
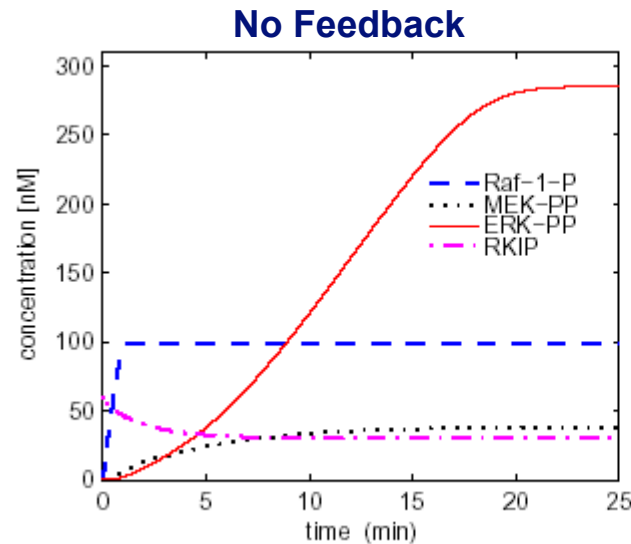


Combination of positive & negative feedback Mathematical Model

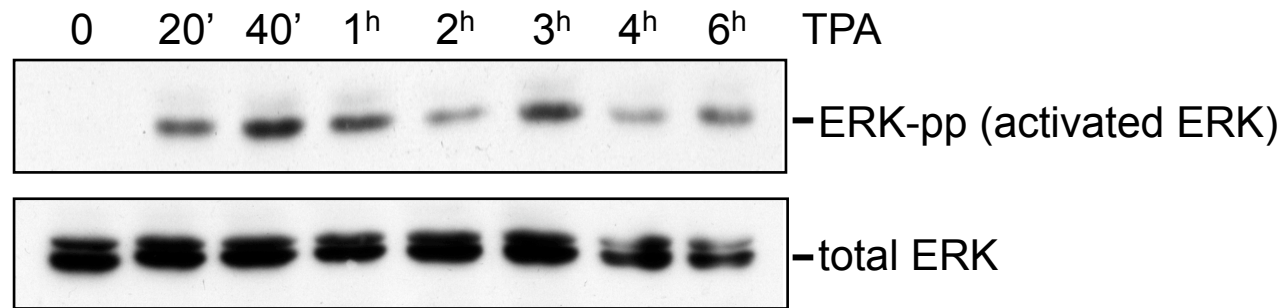


Modeling and Analysis of Two Feedback Loop Dynamics in Ras/Raf-1/MEK/ERK Signaling Pathway
Kwang-Hyun Cho, Sung-Young Shin, Walter Kolch, Olaf Wolkenhauer. ICSB 2004

Combination of positive & negative feedback: Simulation



Combination of positive & negative feedback: Simulation vs. Experimental Data



Western blots COS1 cell lysates

