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Determining Teaching and Research Efficiencies

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In this paper we consider the problem of determining teaching and research efficiencies for university departments concerned with the same discipline. Considering this problem highlights the issue of how to determine efficiencies when resources are shared between different activities, and a non-linear approach to this issue based upon data envelopment analysis is presented. Computational results are given for chemistry and physics departments in the United Kingdom.

Key words: data envelopment analysis, efficiency, universities

INTRODUCTION

Data envelopment analysis (DEA) was first put forward by Charnes *et al.*[1] in 1978 and is used for evaluating the (relative) efficiency of decision-making units via weights attached to input/output measures. The reader new to DEA is referred to the paper by Boussofiane *et al.*[2] and the books by Norman and Stoker[3] and Ganley and Cubbin[4]. Since its introduction DEA has been widely reported in the literature, with a recent bibliography[5] containing some 400 references.

In a previous paper[6] we developed a model, based upon DEA, for comparing university departments (concerned with the same discipline) with respect to their overall efficiency. In this paper we develop a model, based upon DEA, for the joint (simultaneous) determination of the teaching and research efficiencies of university departments. The key to this model is a non-linear approach to apportioning shared resources between these two activities, teaching and research.

Literature survey

In this literature survey, for reasons of space, only relevant work not previously discussed in Beasley[6] is reported.

The use of DEA in relation to university departments is inevitably linked to the wider issue of performance indicators in higher education. Johnes[7] provides an accessible survey of performance indicators in higher education (including DEA). He briefly describes using DEA in conjunction with publications data (books, papers, etc) to derive efficiencies for economics departments in the United Kingdom. This work is reported in greater detail in Johnes and Johnes[8,9].

The reader interested in addressing the wider issue of performance indicators in higher education is referred to the books by Cave *et al.*[10] and Johnes and Taylor[11]. In the next section we briefly review DEA and consider its application to university departments.

DEA REVIEW

In applying DEA to university departments we:

(1) require the input and output measures for each department to be specified;
(2) define efficiency for each department as a weighted sum of outputs (total output) divided by a weighted sum of inputs (total input), where all efficiencies are restricted to lie between zero and one; and

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(3) determine a numeric value for the efficiency of a particular department by maximizing its efficiency through choosing appropriate values for the weights.

These three steps are illustrated below.

Input/output measures

In deciding the input/output measures that we can use to compare university departments we need to have a conceptual view of what are the inputs and outputs for a university department and we also need to consider the data that are actually available.

In order to be consistent with our previous work[6] we shall use exactly the same input/output measures (and data) as used in that work. The eight output measures used in Beasley[6] were:

(1) number of undergraduates (UGs);
(2) number of taught postgraduates (PGs T);
(3) number of research postgraduates (PGs R);
(4) research income—as a proxy for research output in terms of quantity (publications and/or citations);
(5) if a department is rated star (outstanding) at research;
(6) if a department is rated A+ (above average) at research;
(7) if a department is rated A (average) at research;
(8) if a department is rated A− (below average) at research.

The three input measures used in Beasley[6] were:

(1) general expenditure (the majority of this expenditure being on staff salaries);
(2) equipment expenditure;
(3) research income.

The data used by Beasley[6] were those appertaining to the above eight output, and three input, measures for chemistry and physics departments in the United Kingdom for a particular year. Table 1 shows this data for chemistry departments and Table 2 shows this data for physics departments. Note here that this data is publically available via e-mail from OR-Library[12].

Efficiencies

Let:

n = the number of departments,
y_{ik} = the value ($\geqslant 0$) of output measure i ($i = 1, \ldots, 8$) for department k,
x_{jk} = the value ($\geqslant 0$) of input measure j ($j = 1, 2, 3$) for department k,
u_i = the weight ($\geqslant 0$) to be attached to one unit of output measure i,
v_j = the weight ($\geqslant 0$) to be attached to one unit of input measure j,
e_k = the (relative) efficiency of departments k,
ϵ = a very small 'non-Archimedean' number (> 0),

then e_k is defined by:

$$e_k = \left(\sum_{i=1}^{8} u_i y_{ik}\right) \Bigg/ \left(\sum_{j=1}^{3} v_j x_{jk}\right) \qquad k = 1, \ldots, n \tag{1}$$

where:

$$0 \leqslant e_k \leqslant 1 \qquad k = 1, \ldots, n. \tag{2}$$

Maximization

We determine the efficiency (e_p) of department p using the non-linear program:

maximize e_p (3)

subject to: (1) and (2)

$$u_i \geqslant \epsilon \quad i = 1, \ldots, 8 \tag{4}$$

$$v_j \geqslant \epsilon \quad j = 1, 2, 3. \tag{5}$$

This non-linear program can be converted into a linear program using an approach due to Charnes and Cooper[13] and hence easily solved.

TABLE 1. *Data for chemistry departments*

University		General expenditure (£000s)	Equipment expenditure (£000s)	Research income (£000s)	UGs	PGs T	PGs R	Research rating			
								Star	A+	A	A−
k	Value weight	x_{1k} v_1	x_{2k} v_2	x_{3k}, y_{4k} v_3, u_4	y_{1k} u_1	y_{2k} u_2	y_{3k} u_3	y_{5k} u_5	y_{6k} u_6	y_{7k} u_7	y_{8k} u_8
Aston		446	21	183	62	0	37	0	0	0	1
Bath		670	53	288	137	0	43	0	0	1	0
Birmingham		1459	69	288	225	3	63	0	0	1	0
Bradford		613	95	73	92	0	12	0	0	0	1
Bristol		2043	256	1050	253	27	118	1	0	0	0
Brunel		686	46	436	137	18	27	0	0	0	1
Cambridge		2227	620	981	305	0	159	1	0	0	0
City		696	93	354	81	0	31	0	0	0	1
Durham		1027	148	578	187	0	42	0	0	1	0
East Anglia		1155	113	545	126	31	90	0	1	0	0
Essex		620	115	565	76	5	49	0	0	0	1
Exeter		984	138	198	166	0	32	0	0	1	0
Hull		880	78	488	119	9	29	0	0	1	0
Keele		440	51	217	50	0	20	0	0	0	1
Kent		667	281	111	116	0	29	0	0	0	1
Lancaster		685	50	191	92	11	15	0	0	0	1
Leeds		2545	210	763	320	9	82	0	0	1	0
Leicester		919	61	419	173	0	49	0	0	1	0
Liverpool		1259	82	496	195	0	56	0	1	0	0
London											
Birkbeck		734	33	142	46	26	48	0	0	0	1
Imperial		1760	742	1061	167	0	141	1	0	0	0
KQC (Kings)		1487	479	521	240	3	42	0	0	1	0
Q. Mary		1106	170	430	164	3	37	0	1	0	0
R. Hol & Bed		962	131	152	122	0	33	0	0	0	1
Univ. Coll.		1238	67	490	157	4	60	0	1	0	0
Loughborough		1208	89	397	158	26	49	0	0	0	1
Manchester		1920	191	544	268	0	81	0	1	0	0
UMIST		1758	196	1162	237	9	105	0	0	1	0
Newcastle		1211	79	540	157	0	52	0	0	1	0
Nottingham		1409	122	527	263	0	94	0	1	0	0
Oxford		3337	654	1780	707	0	211	1	0	0	0
Reading		908	120	336	162	5	36	0	0	0	1
Salford		1492	127	613	152	18	102	0	0	0	1
Sheffield		1346	78	250	223	0	64	0	1	0	0
Southampton		1620	420	1224	199	2	124	1	0	0	0
Surrey		691	65	407	122	2	27	0	0	0	1
Sussex		1324	144	565	189	13	104	0	1	0	0
Warwick		927	148	359	147	0	43	0	0	1	0
York		947	146	724	236	7	54	0	0	1	0
Aberystwyth		370	32	102	58	1	10	0	0	0	1
Bangor		360	73	122	89	0	7	0	0	0	1
Cardiff		849	32	258	158	3	53	0	1	0	0
Swansea		764	89	317	132	0	31	0	0	1	0
UWIST		560	99	196	100	0	24	0	0	0	1
Aberdeen		1029	126	391	164	2	39	0	0	0	1
Dundee		619	21	136	73	0	13	0	0	0	1
Edinburgh		1381	254	812	292	0	71	0	1	0	0
Glasgow		2253	131	360	354	7	94	0	0	1	0
Heriot-Watt		768	38	324	142	0	25	0	0	0	1
St. Andrews		696	73	408	121	0	29	0	0	0	1
Stirling		421	18	105	57	0	15	0	0	0	1
Strathclyde		1714	112	945	269	15	77	0	1	0	0

TABLE 2. *Data for physics departments*

University		General expenditure (£000s)	Equipment expenditure (£000s)	Research income (£000s)	UGs	PGs T	PGs R	Research rating			
								Star	A+	A	A−
k	Value weight	x_{1k} v_1	x_{2k} v_2	x_{3k}, y_{4k} v_3,u_4	y_{1k} u_1	y_{2k} u_2	y_{3k} u_3	y_{5k} u_5	y_{6k} u_6	y_{7k} u_7	y_{8k} u_8
Bath		528	64	254	145	0	26	0	0	0	1
Birmingham		2605	301	1485	381	16	54	0	1	0	0
Bradford		304	23	45	44	3	3	0	0	0	1
Bristol		1620	485	940	287	0	48	0	1	0	0
Brunel		490	90	106	91	8	22	0	0	0	1
Cambridge		2675	767	2967	352	4	166	1	0	0	0
City		422	0	298	70	12	19	0	0	0	1
Durham		986	126	776	203	0	32	0	0	1	0
East Anglia		523	32	39	60	0	17	0	0	0	1
Essex		585	87	353	80	17	27	0	1	0	0
Exeter		931	161	293	191	0	20	0	0	0	1
Hull		1060	91	781	139	0	37	0	0	0	1
Keele		500	109	215	104	0	19	0	0	0	1
Kent		714	77	269	132	0	24	0	0	0	1
Lancaster		923	121	392	135	10	31	0	0	1	0
Leeds		1267	128	546	169	0	31	0	0	1	0
Leicester		891	116	925	125	0	24	0	1	0	0
Liverpool		1395	571	764	176	14	27	0	1	0	0
London											
Birkbeck		990	83	615	28	36	57	0	0	0	1
Imperial		3512	267	3182	511	23	153	1	0	0	0
KQC (Kings)		1451	226	791	198	0	53	0	0	1	0
Q. Mary		1018	81	741	161	5	29	0	1	0	0
R. Hol & Bed		1115	450	347	148	4	32	0	0	0	1
Univ. Coll.		2055	112	2945	207	1	47	0	1	0	0
Loughborough		440	74	453	115	0	9	0	0	1	0
Manchester		3897	841	2331	353	28	65	1	0	0	0
UMIST		836	81	695	129	0	37	0	0	0	1
Newcastle		1007	50	98	174	7	23	0	0	1	0
Nottingham		1188	170	879	253	0	38	0	0	1	0
Oxford		4630	628	4838	544	0	217	1	0	0	0
Reading		977	77	490	94	26	26	0	0	1	0
Salford		829	61	291	128	17	25	0	0	0	1
Sheffield		898	39	327	190	1	18	0	0	0	1
Southampton		901	131	956	168	9	50	0	1	0	0
Surrey		924	119	512	119	37	48	0	1	0	0
Sussex		1251	62	563	193	13	43	0	0	1	0
Warwick		1011	235	714	217	0	36	0	1	0	0
York		732	94	297	151	3	23	0	0	1	0
Aberystwyth		444	46	277	49	2	19	0	0	1	0
Bangor		308	28	154	57	0	7	0	0	0	1
Cardiff		483	40	531	117	0	23	0	0	0	1
Swansea		515	68	305	79	7	23	0	0	0	1
Aberdeen		593	82	85	101	1	9	0	0	0	1
Dundee		570	26	130	71	20	11	0	0	0	1
Edinburgh		1317	123	1043	293	1	39	0	0	1	0
Glasgow		2013	149	1523	403	2	51	0	1	0	0
Heriott-Watt		992	89	743	161	1	30	0	0	1	0
St. Andrews		1038	82	513	151	13	47	0	0	1	0
Stirling		206	1	72	16	0	6	0	0	0	1
Strathclyde		1193	95	485	240	0	32	0	0	0	1

The key point to note here is that in evaluating the efficiency of department p we choose the weights that *maximize* its efficiency. Conceptually we can regard e_p (when maximized) as the efficiency of department p when compared with its peers.

TEACHING AND RESEARCH EFFICIENCIES

The basic DEA model given above (equations (1)–(5)) gives a value for the overall efficiency of each department. However, how can we determine how efficient each department is at each of its two basic functions, teaching and research? In this section we outline our approach to determining teaching and research efficiencies.

Input/output measures

We need to decide which input/output measures are associated with a department's teaching and which are associated with a department's research. With regard to output measures there would probably be fairly general agreement that:

(a) undergraduates (UGs) and taught postgraduates (PGs T) are associated with teaching; and

(b) all other output measures are associated with research.

However, a problem arises with respect to apportioning input measures to teaching and/or research.

There would probably be fairly general agreement that research income is an input measure associated with research. General expenditure, however, is composed mainly of staff salaries. These staff do both teaching and research—but how much money is being spent on each activity? The author, for example, is paid out of general expenditure but, on his pay slip, it gives just a single figure—i.e. he does not receive two separate payments, one for his teaching activities and one for his research activities.

This question of determining how much of general expenditure is associated with teaching, and how much is associated with research can be phrased as follows:

> How can we determine the proportion of general expenditure associated with teaching and the proportion associated with research?

Similarly:

> How can we determine the proportion of equipment expenditure associated with teaching and the proportion associated with research?

One solution to these problems might be to survey staff/equipment to see what proportion of working time is being spent on teaching and what proportion of working time is being spent on research. We prefer a different solution.

In keeping with the spirit of DEA (if we do not know something let it be a variable whose specific numeric value is determined by an appropriate optimization model) let:

q_1 be the proportion of general expenditure associated with teaching;

q_2 be the proportion of equipment expenditure associated with teaching;

so that:

$1 - q_1$ is the proportion of general expenditure associated with research;

$1 - q_2$ is the proportion of equipment expenditure associated with research

where the limits for q_1 and q_2 are:

$$\epsilon \leqslant q_1 \leqslant 1 - \epsilon \tag{6}$$

$$\epsilon \leqslant q_2 \leqslant 1 - \epsilon \tag{7}$$

in order to ensure that some non-zero proportion of general/equipment expenditure is associated with both teaching and research. Then, letting:

t_k be the (relative) teaching efficiency of department k;

r_k be the (relative) research efficiency of department k;

we have that:

$$t_k = \left(\sum_{i=1}^{2} u_i y_{ik}\right) \Big/ \left(\sum_{j=1}^{2} q_j v_j x_{jk}\right) \qquad k = 1, \ldots, n \tag{8}$$

$$r_k = \left(\sum_{i=3}^{8} u_i y_{ik}\right) \Big/ \left(\sum_{j=1}^{2} (1 - q_j) v_j x_{jk} + v_3 x_{3k}\right) \qquad k = 1, \ldots, n \tag{9}$$

where:

$$0 \leqslant t_k \leqslant 1 \qquad k = 1, \ldots, n \tag{10}$$

$$0 \leqslant r_k \leqslant 1 \qquad k = 1, \ldots, n. \tag{11}$$

Equation (8) defines the teaching efficiency of department k as the weighted sum of its teaching outputs (undergraduates and taught postgraduates, see above) divided by the weighted sum of its teaching inputs (the proportions of general expenditure and equipment expenditure associated with teaching). Equation (9) defines the research efficiency of department k in a similar fashion, but using research outputs and research inputs.

In other words we have the teaching and research efficiencies exactly as we would have done in DEA (using the inputs/outputs associated with teaching/research), except that now these two efficiencies are linked together via the variables q_1 and q_2 representing the shared (apportioned) input measures.

Note here that the advantage of this approach is that it does not require an *a priori* split of expenditure into teaching/research. Instead, such a split is automatically decided, for each department, in a manner that will become apparent below.

Separate DEA maximizations

Consider a specific department p. Having defined our teaching and research efficiencies (t_p and r_p respectively) we need to choose the weights (and proportions) that maximize these efficiencies. At first sight we might consider that these maximum efficiencies can be found via maximizing t_p and r_p separately (i.e. maximize t_p subject to (4)–(11) and maximize r_p subject to (4)–(11)). However we believe that this approach is flawed.

For example, suppose that we were to follow this approach. Then using the data for chemistry departments we have that for department number one (Aston) with a general expenditure of £446 000:

(a) the maximum teaching efficiency is 0.73, which is achieved when the proportion of general expenditure devoted to teaching (q_1) is 0.39, i.e. when teaching related general expenditure ($446000q_1$) is approximately £174 000;

(b) the maximum research efficiency is 1.00 which is achieved when the proportion of general expenditure devoted to teaching (q_1) is 0.99, i.e. when teaching related general expenditure ($446000q_1$) is approximately £442 000.

In other words this department is spending £174 000 on teaching when we are measuring its teaching efficiency but £442 000 on teaching when we are measuring its research efficiency!

The key point here is that it is simply not credible to have two such vastly different figures for the same activity (teaching) depending upon whether we are looking at the department from the point of view of teaching efficiency or from the point of view of research efficiency.

We would note here that this example is not an isolated one but arises because of the underlying nature of the maximization problems solved (maximize t_p subject to (4)–(11) and maximize r_p subject to (4)–(11)). Simply put, in order to maximize teaching efficiency we devote few resources to teaching (i.e. a low q_1, see equation (8)) whilst to maximize research efficiency we devote few resources to research (i.e. a low $(1 - q_1)$, see (9), implying a high q_1).

As this approach of determining t_p and r_p by separate maximization problems is flawed (as detailed above) we prefer the approach detailed below.

Joint DEA maximization

Consider the fraction λ of total (weighted) input resource devoted to teaching for department p. We have that:

$$\lambda = \left(\sum_{j=1}^{2} q_j v_j x_{jp}\right) \Bigg/ \left(\sum_{j=1}^{3} v_j x_{jp}\right) \tag{12}$$

This fraction λ of input resource is 'converted' into (weighted) output with efficiency t_p.

Similarly, the fraction of total (weighted) input resource devoted to research for department p is $(1 - \lambda)$ and this is 'converted' into (weighted) output with efficiency r_p.

The overall efficiency of department p at converting (weighted) input resource into (weighted) output resource is hence:

$$\lambda t_p + (1 - \lambda) r_p \tag{13}$$

and it would seem appropriate that this should be maximized (more on the appropriateness of this objective below).

Hence our model for determining the teaching and research efficiencies (t_p and r_p respectively) of department p is maximize (13) subject to (4) to (12).

In this non-linear model we jointly (simultaneously) determine t_p and r_p, thereby overcoming the problem identified before, associated with separate maximizations.

In fact, we can simplify the above non-linear model (equations (4)–(13)). Substituting for λ, t_p and r_p in the objective function we get:

$$\left[\left(\sum_{j=1}^{2} q_j v_j x_{jp}\right)\Big/\left(\sum_{j=1}^{3} v_j x_{jp}\right)\right]\left[\left(\sum_{i=1}^{2} u_i y_{ip}\right)\Big/\left(\sum_{j=1}^{2} q_j v_j x_{jp}\right)\right] +$$
$$\left[1 - \left(\sum_{j=1}^{2} q_j v_j x_{jp}\right)\Big/\left(\sum_{j=1}^{3} v_j x_{jp}\right)\right]\left[\left(\sum_{i=3}^{8} u_i y_{ip}\right)\Big/\left(\sum_{j=1}^{2}(1 - q_j) v_j x_{jp} + v_3 x_{3p}\right)\right] \tag{14}$$

and after simplification this becomes:

$$\left(\sum_{i=1}^{8} u_i y_{ip}\right)\Big/\left(\sum_{j=1}^{3} v_j x_{jp}\right) \tag{15}$$

which is simply e_p (see equation (1)).

In other words, to determine jointly teaching and research efficiencies we maximize overall efficiency.

Whilst the argument presented above (concerning the choice of objective function (equation (13)) to maximize in order to determine teaching and research efficiencies jointly) is based on the primal DEA program, Mar-Molinero[14] has taken the work presented in this paper and shown that considering the problem from the viewpoint of the dual DEA program also leads to the same objective function.

Note here that had we proposed an approach to determining teaching and research efficiencies jointly, based upon simply adding to the basic DEA model (equations (1)–(5)) the appropriate equations defining teaching and research efficiencies (equations (6)–(11)), we would (after some algebraic manipulation) have ended up with *exactly* the same model (max e_p subject to (4) to (11)) as presented above.

General joint DEA maximization

In the previous section we were concerned with just two component efficiencies (teaching and research) and just two shared input resources (general expenditure and equipment expenditure).

However, it is simple to show that the basic approach given in the previous section, namely to determine component efficiencies we maximize overall efficiency, is quite general. In particular, it is appropriate:

(a) irrespective of the number of different components of efficiency, and

(b) irrespective of which resources (input and/or output) are apportioned between these components.

We believe that this approach will prove to be of value in DEA studies in areas where different functions are being carried out and we desire to determine the efficiency of these functions, yet one (or more) input/output resources are shared across functions and need (somehow) to be apportioned.

MODEL IMPROVEMENT

We believe that it is always vital, when doing a DEA study, to look behind the efficiencies achieved to the actual values of the weights (and proportions) used in achieving those efficiencies. This is because these weights (and proportions) may be such as to throw the credibility of the DEA model into question.

For example, suppose that we take the above model ($\max e_p$ subject to (4) to (11)) and, using the data for chemistry departments, solve it for department number one (Aston). We find that the maximum overall efficiency of this department is one. The weights and proportions associated with this maximum overall efficiency are:

$u_1 = 0.000010$	$u_5 = 1.283839$	$v_1 = 0.442482$	$q_1 = 0.000001$
$u_2 = 0.000047$	$u_6 = 1.295676$	$v_2 = 0.000041$	$q_2 = 0.999999$
$u_3 = 8.880729$	$u_7 = 1.307807$	$v_3 = 53.566406$	
$u_4 = 52.840881$	$u_8 = 1.530801$		

(where we used $\epsilon = 10^{-6}$, and the above figures are to six decimal places). These weights and proportions are simply not credible. For example they imply that:

(a) one research postgraduate is worth $u_3/u_1 = (8.880729/0.000010) =$ approximately 888 000 undergraduates;

(b) of the total general expenditure of £446 000, the amount of general expenditure associated with teaching ($446000q_1$) is less than £1.

Plainly, in achieving a maximum overall efficiency of one for this department, weights and proportions have been used that do not correspond to what, on any sensible view, actually happens in the real world. How then can we improve our model to make it more representative of the actual situation being modelled?

In order to improve our model we introduce more constraints. This addition of constraints involves value judgements. Just as we exercised our judgement in choosing the input and output measures to use so we also exercise our judgement in deciding what are appropriate constraints to add to the model.

Whilst incorporating value judgements into the model can be problematic (e.g. with regard to the calculation of efficient targets) we believe that one advantage of value judgements is that they require the user to think about the relative importance of different factors in an explicit quantitative manner. We illustrate this below.

Output improvements

(1) *Students*

With regard to the relative weight between the three categories of students (undergraduates, taught postgraduates and research postgraduates) our value judgement (as in Beasley[6]) is:

$$u_3 \geq 1.25u_2 \geq 1.25^2 u_1 \tag{16}$$

$$u_3 \leq 2u_1. \tag{17}$$

Equation (16) ensures that the weight associated with a research postgraduate is at least 25% greater than the weight associated with a taught postgraduate and correspondingly for undergraduates. Equation (17) ensures that the weight associated with a research postgraduate is at most twice that associated with an undergraduate.

(2) *Quality*

With regard to the relative weight between the four research rating categories (star, above average, average and below average) our value judgement (as in Beasley[6]) is:

$$u_5 \geqslant 2u_6 \geqslant 2^2u_7 \geqslant 2^3u_8 \quad (18)$$

$$u_5 \leqslant 20u_8. \quad (19)$$

Equation (18) ensures that the weight attached to the research rating of a department is at least twice that attached to a department with a lesser rating. Equation (19) ensures that the weight attached to the research rating of a star department is at most 20 times greater than the weight attached to the research rating of a below average department.

(3) *Minimum proportion*

The entire (weighted) research output ($\sum_{i=3}^{8} u_i y_{ip}$) of a particular department p is made up of three components:
(a) research postgraduates u_3y_{3p},
(b) research quantity u_4y_{4p},
(c) research quality $\sum_{i=5}^{8} u_i y_{ip}$.
Our value judgement is that it is reasonable to expect a department to have a 'minimum proportion' of its total (weighted) research output in each of these three components. For the purposes of this paper we took this proportion to be 10%, hence we have:

$$u_3y_{3p}\Big/\left(\sum_{i=3}^{8} u_i y_{ip}\right) \geqslant 0.1 \quad (20)$$

$$u_4y_{4p}\Big/\left(\sum_{i=3}^{8} u_i y_{ip}\right) \geqslant 0.1 \quad (21)$$

$$\left(\sum_{i=5}^{8} u_i y_{ip}\right)\Big/\left(\sum_{i=3}^{8} u_i y_{ip}\right) \geqslant 0.1. \quad (22)$$

For more about the use of proportion constraints in DEA see Wong and Beasley[15].

Input improvements

(1) *Teaching proportions*

Our value judgement is that it is reasonable to restrict the proportion of general expenditure (q_1), and the proportion of equipment expenditure (q_2), associated with teaching. For the purposes of this paper we used:

$$0.3 \leqslant q_1 \leqslant 0.9 \quad (23)$$

$$0.3 \leqslant q_2 \leqslant 0.9. \quad (24)$$

Equation (23) restricts the proportion of general expenditure associated with teaching to lie between 30% and 90%. Equation (24) similarly restricts the proportion of equipment expenditure associated with teaching to lie between 30% and 90%.

(2) *Minimum proportion*

The entire (weighted) teaching input ($\sum_{j=1}^{2} q_j v_j x_{jp}$) of a particular department p is made up of two components:
(a) general expenditure $q_1v_1x_{1p}$,
(b) equipment expenditure $q_2v_2x_{2p}$.
Our value judgement is that it is reasonable to expect a department to have a 'minimum proportion' of its total (weighted) teaching input in each of these two components. For the purposes of this paper we took this proportion to be 5%, hence we have:

$$q_1 v_1 x_{1p} \Big/ \left(\sum_{j=1}^{2} q_j v_j x_{jp} \right) \geqslant 0.05, \tag{25}$$

$$q_2 v_2 x_{2p} \Big/ \left(\sum_{j=1}^{2} q_j v_j x_{jp} \right) \geqslant 0.05. \tag{26}$$

Similarly, the entire (weighted) research input $(\sum_{j=1}^{2}(1 - q_j)v_j x_{jp} + v_3 x_{3p})$ of a particular department p is made up of three components:
(a) general expenditure $(1 - q_1)v_1 x_{1p}$,
(b) equipment expenditure $(1 - q_2)v_2 x_{2p}$,
(c) research income $v_3 x_{3p}$.
Our value judgement is that it is reasonable to expect a department to have a 'minimum proportion' of its total (weighted) research input in each of these three components. For the purposes of this paper we took this proportion to be 5%, hence we have:

$$(1 - q_1)v_1 x_{1p} \Big/ \left(\sum_{j=1}^{2} (1 - q_j) v_j x_{jp} + v_3 x_{3p} \right) \geqslant 0.05 \tag{27}$$

$$(1 - q_2)v_2 x_{2p} \Big/ \left(\sum_{j=1}^{2} (1 - q_j) v_j x_{jp} + v_3 x_{3p} \right) \geqslant 0.05 \tag{28}$$

$$v_3 x_{3p} \Big/ \left(\sum_{j=1}^{2} (1 - q_j) v_j x_{jp} + v_3 x_{3p} \right) \geqslant 0.05. \tag{29}$$

Complete model

The complete model for determining the teaching and research efficiencies (t_p and r_p respectively) of department p is therefore, maximize e_p subject to (4) to (11) and (16) to (29).

This model is a non-linear program, involving $2n + 14$ variables and $2n + 16$ constraints (ignoring the constraints relating to upper/lower bounds on variables). For the data shown in Tables 1 and 2, n is approximately 50 and this results in a model with a size well within the solution range of modern non-linear programming software.

Results

Table 3 shows the teaching and research efficiencies for chemistry and physics departments in the United Kingdom as determined by our non-linear model (maximize e_p subject to (4) to (11) and (16) to (29)). These results were produced using the GINO[16] non-linear programming software package on a 66 MHz, 8 Mb, 486 PC (with $\epsilon = 10^{-6}$). The total computation time required to produce the results shown in Table 3 was 30 minutes.

CONCLUSIONS

In this paper we presented a model, based upon data envelopment analysis, for jointly determining teaching and research efficiencies for university departments. Constraints based upon value judgements were also included in the model and computational results given.

The key to this model was a non-linear approach to apportioning shared resources between teaching and research. This approach has a wider applicability than just the specific example of university departments dealt with in this paper.

TABLE 3. *Results*

University	Chemistry		Physics	
	Teaching efficiency (%)	Research efficiency (%)	Teaching efficiency (%)	Research efficiency (%)
Aston	68	100	—	—
Bath	86	96	100	90
Birmingham	77	86	57	73
Bradford	58	88	64	91
Bristol	57	100	63	80
Brunel	100	99	75	92
Cambridge	50	100	46	100
City	46	78	n/c	n/c
Durham	71	83	74	83
East Anglia	59	99	48	100
Essex	45	100	62	100
Exeter	66	87	74	70
Hull	59	90	50	69
Keele	46	87	74	83
Kent	64	88	69	77
Lancaster	66	71	59	86
Leeds	55	61	51	79
Leicester	81	93	50	93
Liverpool	67	92	50	78
London—Birkbeck	58	100	32	72
Imperial	34	100	62	100
KQC (Kings)	61	70	49	72
Q. Mary	59	87	66	95
R. Hol & Bed	50	83	48	68
Univ. Coll.	60	95	43	100
Loughborough	67	64	94	85
Manchester	58	84	36	81
UMIST	55	89	59	77
Newcastle	56	86	78	100
Nottingham	78	93	77	76
Oxford	80	93	42	97
Reading	74	70	55	86
Salford	49	63	75	74
Sheffield	75	99	94	54
Southampton	46	100	73	100
Surrey	73	84	70	98
Sussex	63	95	73	89
Warwick	61	84	76	90
York	100	96	77	90
Aberystwyth	67	83	44	98
Bangor	95	74	72	86
Cardiff	100	100	96	99
Swansea	70	86	63	81
UWIST	68	80	—	—
Aberdeen	65	67	63	83
Dundee	68	80	77	85
Edinburgh	81	90	87	81
Glasgow	73	83	82	86
Heriot-Watt	88	79	64	86
St. Andrews	69	84	65	89
Stirling	68	88	36	100
Strathclyde	73	100	81	55

Note: (a) n/c means that efficiencies could not be calculated. This is because this university has zero equipment expenditure (see Table 2) and so the associated non-linear model is infeasible ((26) and (28) cannot be satisfied).

(b) Applying the basic DEA model (equations (1)–(5)) yields an efficiency of one for all universities for both chemistry and physics. This is because we have an output measure with the same numeric value as an input measure (see Tables 1 and 2).

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